Aug. 13, 1957 C. F. KAYAN 2,802,624

APPARATUS FOR CALCULATING HEAT EXCHANGER PERFORMANCE

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FIG. 1

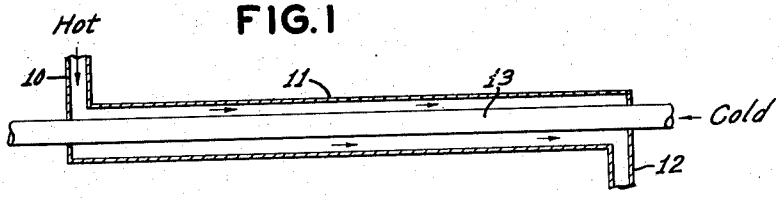


FIG. 2

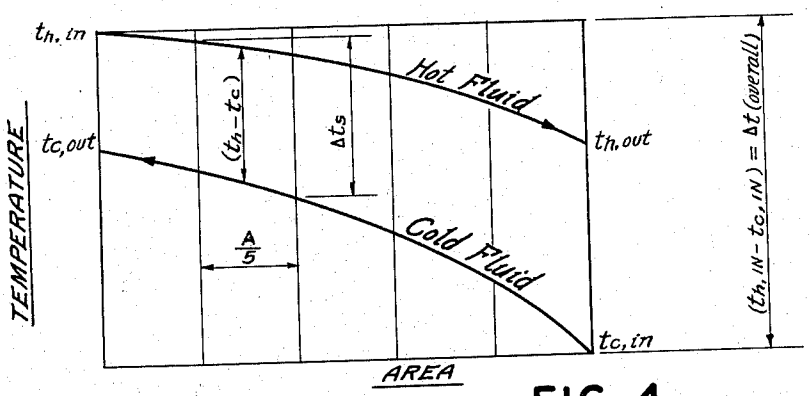


FIG. 3

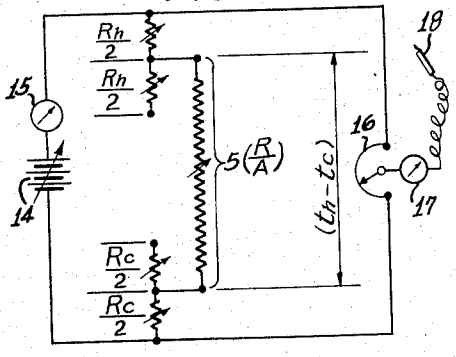
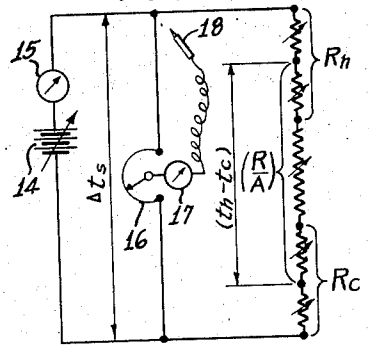


FIG. 4



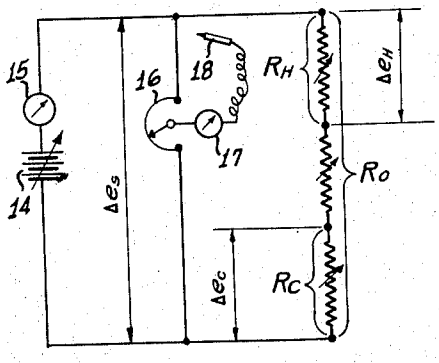


FIG. 5

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APPARATUS FOR CALCULATING HEAT EXCHANGER PERFORMANCE

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FIG. 6

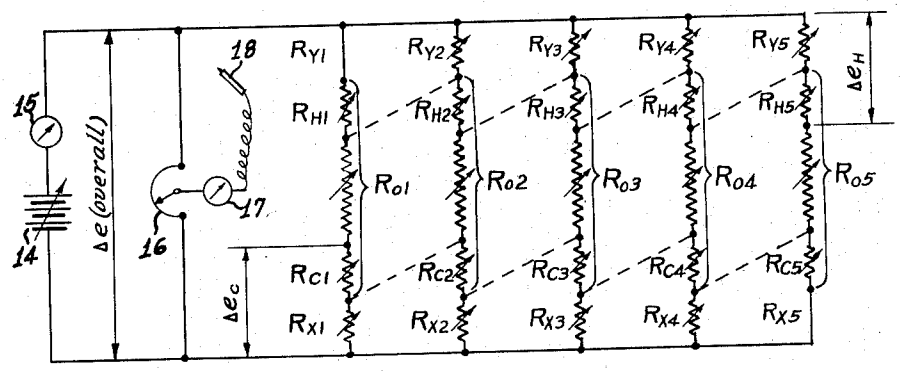


FIG. 7

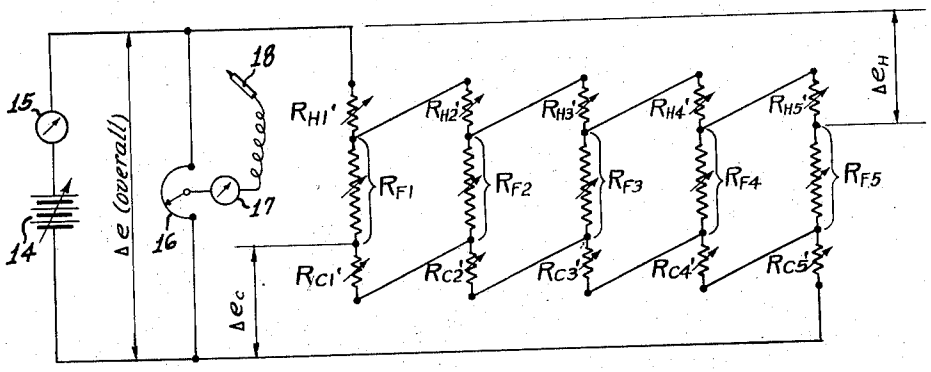
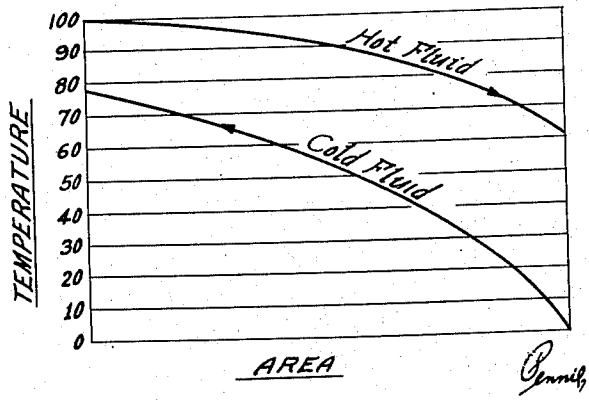


FIG. 8



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2,802,624

APPARATUS FOR CALCULATING HEAT EXCHANGER PERFORMANCE

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Application August 17, 1951, Serial No. 242,231

2 Claims. (Cl. 235—61)

This invention relates to an improved method and apparatus for calculating the performance of industrial heat exchangers for transferring heat from one fluid to another.

The calculation of the performance of heat exchangers by the methods now in use is laborious and time-consuming, and such calculations present a serious problem when it is necessary to take into account varying conditions within the heat exchanger, such as varying specific heat and changing over-all thermal conductance. Varying over-all conductance is a particular problem when the heat transfer process involves radiation. Such would be the case, for example, if one of the fluids were superheated steam in turbulent flow where the conductance on the steam side of the exchanger consists in part of convection and in part of gaseous radiation, and wherein the radiation conductance is distinctly responsive to temperature conditions.

In accordance with the invention an electrical circuit is arranged to simulate the conditions existing within the heat exchanger. In order to do this the electrical circuit comprises a network of electrical resistances which are made proportional to the several thermal resistances (the inverse of thermal conductance) involved in the heat exchange process. The over-all electrical potential difference (voltage difference) of the circuit corresponds to the over-all temperature difference of the heat exchanger, and differences in electrical potential between different parts of the electrical circuit represent differences in temperature between corresponding parts of the heat exchanger. The electrical current resulting from the over-all potential difference, or voltage, impressed upon the circuit represents the over-all heat flow of the heat exchanger.

The thermal resistances of the heat exchanger comprise the resistance to the warming up (heat absorption) of the cold fluid which is a function of the specific heat of that fluid, the resistance to the cooling off (heat releasing) of the hot fluid, a function of the specific heat of that fluid, and the thermal resistance of the heat exchanger structure. The last includes the total series resistance to heat flow between source and receiver fluids. According to the present invention a method of calculating heat exchanger performance has been provided which includes the step of representing the heat absorbing capacity of the cold fluid and the heat releasing capacity of the hot fluid each by means of a resistance to the flow of electrical current. The thermal resistance of the heat exchanger structure is also similarly represented.

The invention will be better understood from the consideration of the accompanying drawings and the following description in greater detail. In these drawings

Fig. 1 is a diagrammatic view of a counterflow fluid-to-fluid heat exchanger;

Fig. 2 is a diagram representing the variations in temperature of the hot and cold fluids as they travel through the heat exchanger;

Fig. 3 is a diagram of an electrical apparatus having a

circuit in simulation of the heat exchanger of Fig. 1, or a section of such exchanger;

Fig. 4 is an illustrative development of Fig. 3;

Fig. 5 is a further illustrative development;

Fig. 6 is a diagram of an apparatus similar to that of Fig. 3, but which simulates the heat exchanger of Fig. 5 divided into five sections;

Fig. 7 is a modification of Fig. 6; and

Fig. 8 is like Fig. 2, but produced by calculation employing the apparatus of Figs. 6 or 7.

The invention will be described and discussed in connection with the calculations for the counterflow type of heat exchanger as that type introduces more problems than the parallel flow type. Referring to Fig. 1, the hot or heat source fluid enters the heat exchanger at **10** and travels toward the right through the heat exchanger shell **11**, leaving by connection **12**. The cold or heat absorbing fluid travels from right to left through the tube bundle, which is represented by the single tube **13**.

The conventional relationships for a two-fluid heat exchanger are given herewith:

$$q=AU\Delta t_m \tag{1}$$

representing the transfer of heat from a hot source fluid to a cold receiver fluid;

$$q=w_c c_c \Delta t_c = E_c \Delta t_c \tag{2}$$

representing the heat involved in the temperature change of the cold fluid;

$$q=w_h c_h \Delta t_h = E_h \Delta t_h \tag{3}$$

representing the heat involved in the temperature change of the hot fluid;

$$q=E_h \Delta t_h = AU\Delta t_m = E_c \Delta t_c \tag{4}$$

where

q=time-rate of gross heat exchange, B. t. u./hr.

A=total exchanger area, ft.^{2}

U=over-all heat transfer conductance between source and receiver fluids, B. t. u./(hr. sq. ft. per ° F.)

Δt_m=logarithmic mean temperature difference (ideally), °F.

$$=\frac{(t_{h\ in}-t_{c\ out})-(t_{h\ out}-t_{c\ in})}{ln\left(\frac{t_{h\ in}-t_{c\ out}}{t_{h\ out}-t_{c\ in}}\right)}$$

with t_h and t_c representing hot and cold substance temperatures, distinguished as to "in" and "out" conditions

w_c=flow rate of cold substance, lb./hr.

c_c=specific heat of cold substance, B. t. u./(lb. ° F.)

E_c="water equivalent" of cold substance, B. t. u./(hr. ° F.)

w_h=flow rate of hot substance, lb./hr.

c_h=specific heat of hot substance, B. t. u./(lb. ° F.)

E_h="water equivalent" of hot substance, B. t. u./(hr. ° F.)

The temperature relationships may also be expressed as follows:

$$\left(\frac{t_{h\ in}-t_{c\ out}}{t_{h\ out}-t_{c\ in}}\right)=e^{AU(1/E_h-1/E_c)} \tag{5}$$

where

e=the base of natural logarithms

Analysis of steady-state heat flow through complex structures has been accomplished extensively by means of electrical analogy (1, 2, 3)[2]. The field of usefulness for the electrical analogy approach may be greatly extended through application to heat exchanger steady-state proc-

esses. According to the resistance concept of heat transfer, Equation 1 may be written:

$$q=AU\Delta t_m=A\Delta t_m/R=\Delta t_m/(R/A) \quad (6)$$

where

R=1/U, °F./(B. t. u. sq. ft. hr.)
=thermal resistance (per sq. ft. of heat transfer area), and representing the total series resistance to heat flow between source and receiver fluids; in simplest form comprising the resistance between the hot fluid and the intervening wall, the wall resistance and the resistance between the wall and the cold fluid; R/A is the net resistance for area A

The concept of equivalent thermal resistance may similarly be applied to the heat exchanger fluids themselves.

Thus, Equations 2 and 3 may be rewritten:

$$q=E_c\Delta t_c=\Delta t_c/R_c \quad (7)$$
$$q=E_h\Delta t_h=\Delta t_h/R_h \quad (8)$$

where

$R_c=1/E_c$, °F./(B. t. u. hr.)
=thermal equivalent corresponding to the cold substance water equivalent, representing the resistance to the warming-up operation
$R_h=1/E_h$, °F./(B. t. u. hr.)
=thermal resistance corresponding to the hot substance water equivalent, representing the resistance to the cooling-off operation

Thus, in accordance with this extension of the conventional heat transfer resistance concept, Equation 4 may now be shown to include "heat exchange" resistance:

$$q=\Delta t_m/(R/A)=\Delta t_c/R_c=\Delta t_h/R_h \quad (9)$$

Therefore

$$\Delta t_m=q(R/A) \quad (10)$$
$$\Delta t_c=qR_c \quad (11)$$
$$\Delta t_h=qR_h \quad (12)$$

An electrical network may be set up to simulate heat transfer in a fluid-to-fluid heat exchanger, illustratively, the counterflow heat exchanger indicated in Fig. 1. Herein electrical resistances are chosen proportional to thermal resistances represented in Equations 10, 11, and 12, and electrical potentials are taken as representative of temperatures. To facilitate operations, the over-all temperature difference (t_h in $-t_c$ in) which is represented by an over-all voltage difference, is considered as 100 percent; thus intermediate potential differences are indicated as some fraction of the over-all 100 percent. The equivalent over-all heat flow thus may be determined by means of the net thermal resistance represented by the net electrical resistance of the simulating circuit. Total current flow on the basis of the over-all impressed direct current voltage may, on the basis of appropriate circuit scale factors, indicate the over-all heat flow rate. Thus the electrical analogy analysis will permit the prediction of thermal performance, but, it should be noted, only in terms of the assumed basic data. It is recognized that calculations could likewise be directly carried out on the basis of the corresponding thermal resistance circuit, but, as will be evident later, electrical experimental operations often are more readily executed.

For the purposes of simplicity, the countercurrent heat exchanger of Fig. 1 has been considered as divided into five equal-area sections. It will be apparent that any number of sections to suit the desired accuracy may be employed, i. e., the pattern of the circuit equations makes it possible to set them up for n sections. Fig. 2 indicates the five sections.

For any one section, linear change in temperature is assumed, and consequently, linear change in electrical potential in the equivalent electrical circuit. The temperature difference at the midpoint of the section is

representative of the heat flow through the section (arithmetic mean temperature difference), that is, as due to the thermal resistance for the section of area $A/5$ for a system of five sections and gross area A. Thus the contribution to total heat flow for this section is given by

$$\Delta q=(t_h-t_c)/[R/(A/5)]=(t_h-t_c)/[5(R/A)]$$

This heat flow is likewise represented in the change in temperatures for both the hot and the cold fluids. The change in temperature for the fluids will be given by Equations 11, $\Delta t_c=(\Delta q)R_c$, and 12, $\Delta t_h=(\Delta q)R_h$. Circuitwise, these relationships may be incorporated into a network arrangement as given in Fig. 3.

Since the total change in the fluid temperatures for the section is desired, and since the heat transfer energy coincides with the heat exchange energy for the fluids, the circuit may be modified as given in Fig. 4, shown for thermal conditions, and Fig. 5, for electrical conditions. Δt_s represents the over-all difference in temperature for one section, and Δe_s the corresponding over-all difference in electrical potential for the simulating electrical analogy circuit.

Fig. 4 thus represents the basic element in the translation to an electrical analogy circuit. Here Δe_s represents the over-all potential difference, $R_H=$ and R_C the equivalent electrical resistance corresponding to the heat exchange resistances $R_h=1/E_h$ and $R_c=1/E_c$, and R_o represents the total over-all resistance of the circuit. If R_U represents the equivalent electrical resistance corresponding to the heat transfer net thermal resistance $R/A=1/(AU)$, then if n sections are used, the equivalent electrical resistance corresponding to the section heat transfer resistance will be nR_U, i. e., for a five-section system, $5R_U$. Thus, $R_o=5R_U+(R_H+R_C)/2$. Potential differences Δe_H and Δe_C represent, to scale, the temperature changes for the hot and cold fluids, respectively.

In Figs. 3, 4, 5, 6 and 7 a source of adjustable electrical potential is indicated at 14 and an ammeter at 15. Also a slide-wire potentiometer is shown at 16 together with a galvanometer 17 and a probing contact 18. By means of this device the relative potential of any desired point in the resistance network can be quickly determined.

If both the heat transfer and the heat exchange fluid thermal resistances are regarded as constant over the entire area of the exchanger (constant U and constant fluid specific heats), the five-section electrical network for the heat exchanger simulation may be shown as in Fig. 6.

The different electrical resistances R_y and R_x represent ballast resistances to compensate for the successive and additive "fluid" potential differences accomplished in each section, such that the total fluid temperature changes will be simulated by Δe_H and Δe_C for the hot and cold fluids, respectively. The total flow is represented by the total circuit current I, representing the sum of the separate current contributions from each of the five sections. The dotted lines in the figure, running from section to section, connect points of equal potential, in accordance with the additive Δe principle cited above.

In accordance with the stipulation of constant U and constant fluid specific heats, the following conditions are in force:

$$R_{H1}=R_{H2}=R_{H3}=R_{H4}=R_{H5}=R_H$$
$$R_{C1}=R_{C2}=R_{C3}=R_{C4}=R_{C5}=R_C$$
$$R_{o1}=R_{o2}=R_{o3}=R_{o4}=R_{o5}=R_o$$

Values for R_y and R_x for the different sections are given by the following relationships, in which

$$P=N/M \quad (13)$$
$$Q=M/N \quad (14)$$

where

$$M=R_o-R_H$$
$$N=R_o-R_C$$

Ballast resistances R_y:

$$R_{y1}=0$$
$$R_{y2}=R_H(P)$$
$$R_{y3}=R_H(P+P^2) \quad (15)$$
$$R_{y4}=R_H(P+P^2+P^3)$$
$$R_{y5}=R_H(P+P^2+P^3+P^4)$$

Ballast resistances R_x:

$$R_{x5}=0$$
$$R_{x4}=R_C(Q)$$
$$R_{x3}=R_C(Q+Q^2) \quad (16)$$
$$R_{x2}=R_C(Q+Q^2+Q^3)$$
$$R_{x1}=R_C(Q+Q^2+Q^3+Q^4)$$

In accordance with the analogy relationships of Equations 15 and 16, the five-section network corresponding to Fig. 6 has been calculated. The following basic information has been assumed: E_C=500 B. t. u./(hr. ° F.), E_H=1000 B. t. u./(hr. ° F.), AU=1000 B. t. u./(hr. ° F.), i. e., combining A and U. Proportional electrical values are as follows: R_C=200 ohms, R_H=100 ohms, R_U=100 ohms, R_o=5(100)+(100+200)/2=650 ohms. The circuit values are given in Table 1:

Table 1

Section	1	2	3	4	5
R_y	0	82	149	204	249
R_o	650	650	650	650	650
R_x	1,350	904	542	244	0
Total R (ohms)	2,000	1,636	1,341	1,098	899

Relative potential values for each section are calculated and shown in Table 2, indicative of the fluid temperatures. The end values of the hot and cold fluids are 1.00 and 0, respectively.

Table 2

Section	1	2	3	4	5
R_x+R_C	1,550	1,104	742	444	200
$\Delta e_C/\Delta e$ over-all	0.775	0.675	0.553	0.405	0.222 0 (end)
R_y+R_H	100	182	249	304	349
$\Delta e_H/\Delta e$ over-all	0.05	0.111	0.185	0.277	0.389
$(1-\Delta e_H)/\Delta e$ over-all 1.000	0.950	0.889	0.815	0.723	0.611

The combined resistance of the network is 258 ohms.

The circuit of Fig. 6 may be replaced by an equivalent circuit (Fig. 7) in which the ballast resistances R_y and R_x are eliminated. The values of R_C and R_H are modified so that they will in turn produce the identical results as to potential drop as before. Each section is electrically joined to the adjoining sections. The circuit elements of R_C and R_H beyond R_{C1} and R_{H5}, respectively, carry extra current in addition to those of their own sections. A separate resistance element R_F (i. e., $R_{F1}-R_{F5}$) will be identified (Fig. 7) as herewith indicated: $R_F=R_o-(R_H+R_C)$. It is to be noted that R_U, R_C and R_H are basically constant, and therefore

$$R_{F1}=R_{F2}=R_{F3}=R_{F4}=R_{F5}=R_F$$

Values for the modified versions of R_H and R_C may be computed from the following relationships, with $P=N/M$, and $Q=M/N$, as previously shown in Equations 13 and 14.

$R_{C1'}=R_C$, i. e., original value
$R_{C2'}=R_C/(1+P)$
$R_{C3'}=R_C/(1+P+P^2)$ (17)
$R_{C4'}=R_C/(1+P+P^2+P^3)$
$R_{C5'}=R_C/(1+P+P^2+P^3+P^4)$

$R_{H5'}=R_H$, i. e., original value
$R_{H4'}=R_H/(1+Q)$
$R_{H3'}=R_H/(1+Q+Q^2)$ (18)
$R_{H2'}=R_H/(1+Q+Q^2+Q^3)$
$R_{H1'}=R_H/(1+Q+Q^2+Q^3+Q^4)$

On the basis of the preceding equations, a new network corresponding to Fig. 7 was set up. The values for this network, based on the data as used for the ballast network case reported in Tables 1 and 2, are shown in Table 3.

Table 3

Section	1	2	3	4	5
$R_{H'}$	12.9	18.1	27.0	4,500	100.0
R_F	350.0	350.0	350.0	350.0	350.0
$R_{C'}$	200.0	110.0	80.2	65.8	57.3

Apparatus corresponding to Fig. 7 as well as apparatus corresponding to Fig. 6 have been constructed and used in heat exchanger calculations. The experimental results obtained from electrical measurements on the network of Fig. 7, adjusted as to values of $R_{H'}$ and $R_{C'}$, follow in Table 4, and are to be compared with those cited in Table 2. It is to be noted that the agreement is acceptable.

Table 4

Section	1	2	3	4	5
$\Delta e_C/\Delta e$ over-all	0.770	0.673	0.556	0.405	0.218 0 (end)
$1-\Delta e_H/\Delta e$ over-all 1.00 (end).	0.952	0.891	0.817	0.724	0.606

The combined resistance of the network measured 257 ohms.

Corresponding values for the temperatures of the hot and cold fluids, calculated by use of the apparatus of the invention and based on conventional algebraic heat exchanger relationships for the same conditions as represented by Equation 5, are shown in Table 5.

Table 5

Section	1	2	3	4	5
$\Delta e_C/\Delta e$ over-all	0.775	0.676	0.550	0.405	0.227 00 (end)
$1-\Delta e_H/\Delta e$ over-all 1.000 (end).	0.951	0.889	0.815	0.726	0.612

It is to be noted that Tables 2, 4, and 5 show good agreement, it being recognized that Tables 2 and 4 are derived from a lumped circuit arrangement. Fig. 8 represents the results of Table 4 on a temperature basis.

Values for Figs. 6 and 7 (Tables 1 and 3) were set up in terms of constancy of values for R_C and R_H, as well as for R_U, for all of the different sections. It is, however, to be recognized that the corresponding thermal values for R_C, R_H and R (i. e., U), are themselves a function of temperature conditions, and hence, with changes indicated in temperatures, not constant. Since, for example, $R=1/U$, and is, as indicated, composed of the hot-fluid component, the wall component, and the cold-fluid component, R will be subject to the variations in these component resistances as dependent on temperature. Thus

$$R=R_{Fh}+R_W+R_{Fc} \quad (19)$$

where R_{Fh} and R_{Fc} are the resistance components corresponding to the boundary conductances of the hot and cold fluid, respectively, and Rw is the resistance component of the intervening wall.

The boundary conductances of the fluids may themselves be composite. For example, if the hot fluid were high pressure superheated steam, then the total boundary conductance could be stated as

$$h_{Fh}=h_c+h_r \quad (20)$$

where

h_{Fh}=composite boundary conductance, B. t. u./(hr. sq. ft. ° F.)

h_c=boundary conductance due to convection (forced), B. t. u./(hr. sq. ft. ° F.)

h_r=equivalent conductance of steam gaseous radiation corresponding to the existent temperature conditions, in B. t. u./(hr. sq. ft. ° F.)

Accordingly, h_{Fh} and R_{Fh} $(=1/h_{Fh})$ will be particularly subject to the temperature conditions, hence variable throughout the traverse of the area of the heat exchanger. R_c and R_h in turn depend on the realized values of specific heat, i. e., the existing temperatures.

The electrical analogy approach lends itself especially to the solution of the heat exchanger problem with varying properties, such as for example, introduced through the variation of the gaseous radiation cited above, etc. Accordingly, the solution of the five-section counter-current heat exchanger problem has been developed to deal with the varying properties, using the electrical analogy approach.

Based on the ballast circuit of Fig. 6, values for R_x and R_y throughout the five-section circuit have been set up, noting that R_H, R_c, and R_o are no longer constant, but are considered to change from section to section. Let it be noted that $R_{o1}=5R_{U1}+(R_{C1}+R_{H1})/2$, $R_{o2}=5R_{U2}+(R_{C2}+R_{H2})/2$, etc.

Values for P and Q change as the sections are traversed. The values for M and N change from section to section. Thus $M_1=R_{o1}-R_{H1}$, $M_2=R_{o2}-R_{H2}$, etc. $N_1=R_{o1}-R_{C1}$; $N_2=R_{o2}-R_{C2}$, etc.

Thus

$$P_1=N_2/M_1;\ P_2=N_3/M_2;\ P_3=N_4/M_3;\ P_4=N_5/M_4 \tag{21}$$

$$Q_1=M_1/N_2;\ Q_2=M_2/N_3;\ Q_3=M_3/N_4;\ Q_4=M_4/N_5 \tag{22}$$

The ballast resistance values for R_x and R_y at the different sections are shown

$$\begin{aligned}
&R_{y1}=0\\
&R_{y2}=R_{H1}(P_1)\\
&R_{y3}=R_{H2}(P_2)+R_{H1}(P_1P_2)\\
&R_{y4}=R_{H3}(P_3)+R_{H2}(P_2P_3)+R_{H1}(P_1P_2P_3)\\
&R_{y5}=R_{H4}(P_4)+R_{H3}(P_3P_4)+R_{H2}(P_2P_3P_4)+R_{H1}(P_1P_2P_3P_4)
\end{aligned} \tag{23}$$

$$\begin{aligned}
&R_{x5}=0\\
&R_{x4}=R_{C5}(Q_4)\\
&R_{x3}=R_{C4}(Q_3)+R_{C5}(Q_4Q_3)\\
&R_{x2}=R_{C3}(Q_2)+R_{C4}(Q_3Q_2)+R_{C5}(Q_4Q_3Q_2)\\
&R_{x1}=R_{C2}(Q_1)+R_{C3}(Q_2Q_1)+R_{C4}(Q_3Q_2Q_1)+R_{C5}(Q_4Q_3Q_2Q_1)
\end{aligned} \tag{24}$$

Similarly as for the constant properties case represented by Fig. 7, circuit values may be calculated for the electrical analogy network with ballast resistances R_x and R_y eliminated, and accounting for variable properties. Herein $R_{F1}=5R_{U1}-(R_{H1}+R_{C1})/2$, $R_{F2}=5R_{U2}(R_{H2}+T_{C2})/2$, etc. $R_{o1}=5R_{U1}+(R_{H1}+R_{C1})/2$, $R_{o2}=5R_{U2}+(R_{H2}+R_{C2})/2$, etc.

$$\begin{aligned}
&R_{C1'}=R_{C1}\\
&R_{C2'}=R_{C2}/(P_1+1)\\
&R_{C3'}=R_{C3}/(P_1P_2+P_2+1)\\
&R_{C4'}=R_{C4}/(P_1P_2P_3+P_2P_3+P_3+1)\\
&R_{C5'}=R_{C5}/(P_1P_2P_3P_4+P_2P_3P_4+P_3P_4+P_4+1)\\
&R_{H5'}=R_{H^5}\\
&R_{H4'}=R_{H4}/(Q_4+1)\\
&R_{H3'}=R_{H3}/(Q_3Q_4+Q_3+1)\\
&R_{H2'}=R_{H2}/(Q_2Q_3Q_4+Q_2Q_3+Q_2+1)\\
&R_{H1'}=R_{H1}/(Q_1Q_2Q_3Q_4+Q_1Q_2Q_3+Q_1Q_2+Q_1+1)
\end{aligned} \tag{25}$$

In the case of the circuits designed for variable properties, the procedure is to start with estimated values for the different properties, and on determination of resultant temperatures on this basis, to correct the different values accordingly. Repeated corrections can be introduced until the accuracy requirement is satisfied.

It is to be noted that if $R_H=0$, the resultant solution represents the heat exchanger case of constant temperature source, such as is represented by the ideal condenser

or feed water heater case. If $R_C=0$, the resultant solution represents the heat exchanger case of constant temperature receiver, such as is found in a refrigeration brine cooler. It is believed that the indicated method of electrical analogy can likewise be successfully applied to non-orthodox cases, such as cross-flow exchangers.

The following specific example is illustrative of the application of the method and apparatus of the invention. Let us assume that 1100 lb./hr. of liquid, specific heat 0.91, are to be cooled from an initial temperature of 100° F., by brine, 400 lb./hr., initial temperature=0° F., specific heat=1.25, in a countercurrent heat exchanger of 10 sq. ft. heating surface area, for which U the coefficient of heat transfer, under the flow circumstances, is 100. (*a*) For constant specific heats and constant value of U, show the final outlet temperatures of the two fluids, the variation of the fluid temperature throughout the exchanger, and the total amount of heat transferred per hour. (*b*) What would be the outlet temperatures for 200° F. initial hot fluid temperature, and 60° F. initial cold fluid temperature.

Here

$$E_h=w_{hch}=1100\times0.91=1000 \text{ B. t. u./° F. hr.}$$
$$E_c=w_{cc_c}=400\times1.25=500 \text{ B. t. u./° F. hr.}$$
$$AU=10\times100=1000 \text{ B. t. u./° F. hr.}$$

(as shown in column 5, paragraph 1).

A five-section network may be set up to represent the heat flow in the exchanger by the method of electrical analogy.

$$R_H=1/E_h=1/1000=.001$$
$$R_C=1/E_c=1/500=.002$$
$$R_U=1/AU=1/1000=.001$$

These represent the relative basic resistances.

In accordance with the analogy relationships of Equations 15 and 16, the five section network corresponding to Fig. 6 has been calculated. For convenience in use of resistor values, the different resistances shown above are represented in ohms, as multiplied by a factor of 100,000. Thus, with the relative values retained, proportional electrical values are as follows: $R_H=100$ ohms; $R_C=200$ ohms; $R_U=100$ ohms, and $R_o=5(100)+(100+200)/2=650$ ohms.

The five-section circuit values are given in Table 1. The measured over-all resistance of the network constructed in accordance with the values of Table 1 measures 258 ohms. By means of the potential dividing slide-wire device **16** and galvanometer **17**, the relative potential at various points in the network with respect to the over-all (100%) may be determined. These values are shown in Table 2.

The thermal resistance corresponding to the measured over-all resistance of 258 ohms is 258/100,000=.00258.

For 100 volts over-all potential corresponding to a temperature difference of 100−0=100° F., the current flow shows .388 amps. corresponding to 38,800 B. t. u./hr. For 140 volts, corresponding to an over-all temperature difference of 200−60=140° F., the current flow shows .542 amps. corresponding to 54,200 B. t. u./hr.

The temperature of the fluids at any point may be established from the measured relative potential values obtained from the circuit. Thus, the temperature may be calculated

$t_{\text{Fluid}}=t_{\text{cold, initial}}+C\Delta t_{\text{over-all}}$ where C=relative potential value, with $C=0$ taken at the low or "cold" side of the circuit, and $\Delta t_{\text{over-all}}$=difference between incoming hot fluid and initial cold fluid, $t_{\text{cold, initial}}$.

Thus, based on Table 2 and $t_{c,\text{ initial}}=0°$ F., the outgoing cold fluid temperature=$t_{c,\text{ out}}=0+.775\times100=77.5°$ F. The outgoing hot fluid temperature=$0+0.611\times100=61.1°$ F.

In a similar fashion, the temperatures of the fluids at the different sections may readily be calculated.

Fig. 7 shows the plot of fluid temperatures as calculated above.

The equivalent values for the second case with 140° F. over-all temperature difference are readily calculated. Thus, the outgoing cold fluid temperature will be: $t_{Fluid}=60+.775\times140=60+108.2=168.2°$ F. The outgoing hot fluid temperature $=60+.611\times140=145.5°$ F.

In place of the five-section network as used above and corresponding to Fig. 6, the equivalent electrical circuit corresponding to Fig. 7 may be used with similar results to the above. In addition, although these two previous networks of Figs. 6 and 7 are predicated on constant values of R_H, R_C and R_U, since average specific heats are dependent on the temperature ranges encountered, and the over-all conductance U may change with temperature conditions, the network circuit values corresponding to Fig. 6 may be modified on the basis of Equations 23 and 24, and corresponding to Fig. 7, by Equations 25 and 26.

These modifications permit the analogy network "calculator" of the invention to provide analytical results which would be extremely difficult to establish by orthodox calculation methods. For example, the calculations just described require only a few minutes, and if it is desired to ascertain the effect of changes in specific heat or of the heat exchanger conductance, re-calculation using the new values of these factors can be done in a similarly short space of time.

I claim:

1. In an apparatus for calculating the performance of a heat exchanger transferring heat between two fluids which are continuously changing in temperature as they pass through the heat exchanger but wherein, at any given point in the apparatus, the temperatures of the fluids and the intervening separating wall are substantially constant, a plurality of electrical circuits each representing a section of said heat exchanger, each of said circuits comprising a plurality of electrical resistances, said resistances corresponding respectively to the warming-up of the cold fluid passing through said exchanger, the structural thermal resistance of the heat exchanger and the thermal resistance to the cooling-off of the hot fluid passing through said exchanger, the resistances in each of said circuits being arranged in series connection, each of said resistances being adjustable, said plurality of circuits being connected in parallel so as to cause the electrical currents flowing in them to be added together and thus represent the heat exchanger as a whole, means for measuring the total current flowing in said parallel circuits so as to determine the total heat flowing between said fluids, and an adjustable potential source for impressing on said parallel circuits an electrical potential corresponding to the over-all temperature difference in the particular heat exchanger to be calculated.

2. Heat exchanger calculating apparatus as set forth in claim 1 wherein means are provided for determining the electrical potential of any point in any of said circuits as an indication of the temperature of the corresponding part of the heat exchanger.

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