Abstract: A power supply system comprising a plurality of power generation units connected to each other via power lines and comprising at least one control unit adapted to adjust an active power generated by each power generation unit before occurrence of at least one load step and/or generation step within said power supply system to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system to a stable post-load/generation step operating point of said power supply system.
Description

Method and apparatus for a load step robust unit commitment of power generation units in a power supply system

The invention relates to a method and apparatus for a load step robust unit commitment of power generation units in a power supply system and in particular to a high voltage power supply system.

An electrical power supply system or electrical grid is a network for delivering electricity or current from suppliers to consumers. A power supply system comprises power generation units that produce electrical power and power transmission lines that transport the generated electrical power from the power generation units to loads. There are different kinds of power generation units and loads. For instance, power generation units can comprise as a high voltage power supply system hydroelectric plants, nuclear plants, coal plants as well as medium-sized power plants. A high voltage power supply system comprises a transmission grid of power lines for distributing power over long distances. Medium to low voltage power supply systems comprise a distribution grid to distribute power to consumers. Power generation units, such as solar farms or wind farms, can also be connected to the distribution grid. The transmission network of the high voltage power supply system transports electrical power over long distances. Upon arrival at a substation the electrical power can be stepped down from a transmission voltage level to a distribution voltage level. Further, upon arrival at a service location, the electrical power can be stepped down again from the distribution voltage level to a required service voltage.
In a power supply system, an optimal power generation assignment to different power generation units satisfying the power demand within the system is desired. As the supply and demand uncertainty increases due to the integration of variable generation resources such as wind power or solar energy, unit commitment becomes an even more important task in electric power system operations. Unit commitment is a task performed by system operators in a deregulated power supply system and has the objective to find a unit commitment schedule that minimizes the commitment and dispatch requirements of meeting a forecasted system load, taking into account various physical, intertemporal constraints for generating resources, power transmission as well as system reliability requirements. The method of the present invention takes in addition load/generation step robustness constraints explicitly into account. Usually, the task of unit commitment is performed by a control unit executing a unit commitment algorithm taking into account the physics of the power supply grid incorporating power flow equations as constraints. These power flow constraints guarantee that the assigned power generation leads to a power supply system with a locally stable equilibrium. With the locally stable equilibrium, the power supply system can show a predetermined preferred behaviour. This locally stable equilibrium forms an operating point of the power supply system. However, it is not clear which power supply system states of the power supply system eventually do converge to this locally stable equilibrium, i.e. how much the current power supply system behaviour may be different from a preferred behaviour such that this preferred behaviour is eventually achieved or reached. This region around the equilibrium, i.e. the behaviour from where the power supply
system behaviour eventually reaches the preferred behaviour is also called the region of attraction.

The determination of this region of attraction is important for the power supply system control, especially if the power supply system is affected by a large load or generation step. A large generation step can be a result of fluctuating power generation of renewable energy resources like wind and solar power. The load step can occur when a load connected to the power supply system is switched on or off. A wind farm having a plurality of wind power generation units can also lead to load steps, for instance if the wind driving the blades of the power generation units diminishes briskly. If the load or generation steps are large they do heavily influence the equilibrium of the power supply system, i.e. the preferred behaviour of the power supply system.

Accordingly, it is an object of the present invention to provide a method and apparatus for a load step robust unit commitment of the power generation units within the power supply system.

This object is achieved by a power supply system comprising the features of claim 1.

Accordingly, the invention provides, according to a first aspect, a power supply system comprising a plurality of power generation units connected to each other via power lines and comprising at least one central control unit adapted to adjust an active power generated by each power generation unit before occurrence of at least one load step and/or generation step within said power supply system to provide an admissible transition from a stable pre-load/generation step operating
point of said power supply system to a stable post-load/generation step operating point of said power supply system.

In a possible embodiment of the power supply system according to a first aspect of the present invention, the power generation units form nodes of a power supply network which comprises power lines connecting the power generation units and a plurality of loads.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, the power generation unit of a node comprises a synchronous generator having an associated reference power, damping and inertia.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, for each node of said power supply network, the phase dynamics of each generator follow the differential equations:

\[ M_i \ddot{\theta}_i + D_i \dot{\theta}_i = \tilde{p}_{i0} - P_i + \sum_{j \in N_i} b_{ij} U_{ij} \sin(\theta_i - \tilde{\theta}_j) \]

\[ Vi = \{1...\} \]

wherein \( \tilde{p}_{i0} \) is the reference power of the power generator of node \( i \),
\( D_i \) is the damping of said power generator at node \( i \),
\( M_i \) is the inertia of said power generator at node \( i \),
\( P_i \) is a load at node \( i \),
\( b_{ij} \) is the susceptance of a power line connecting said node with a neighboring node \( j \) of said power supply network,
\( u_i, u_j \) are voltages at node \( i \) and its neighboring node \( j \) connected via said power line, \\
\( \theta_i, \theta_J \) are phase angles at node \( i \) and its neighboring node \( j \) connected via said power line.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, a bound of the maximum distance between the phase difference \( \theta_{ij}^{x(i)} \) at the stable pre-load/generation step operating point before occurrence of the load/generation step and the phase difference \( \theta_{ij}^{x(i)} \) at the stable post-load/generation step operating point after occurrence of the load/generation step is given by:

\[
\max_{(i,j) \in \mathcal{E}} |\Delta \Theta_{ij}^x| \leq 2 \arcsin \left( \frac{\left\| B^T L^T \Delta p \right\|_2}{2 \sqrt{1 - \max \left\{ \left\| B^T L^T p \right\|^2, \left\| B^T L^T (p + \Delta p) \right\|^2 \right\}} \right)
\]

wherein \( \Delta \Theta_{ij}^x \) is the difference between \( \theta_{ij}^{x(i)} \) and \( \theta_{ij}^{x(2)} \), \\
wherein \( B^T \) is an incidence matrix of the network topology of said power supply network, \\
\( \mathcal{E} \) is a pseudo-inverse of the power supply network's Laplacian matrix \( L = B - \text{diag}(a_i) - B^T \), \\
wherein \( a_i = b_i - u_j - u_j \), \\
where \( p \) is the vector of \( p_i(\bar{p}_0, \bar{\pi}_0) = \bar{p}_0 - \bar{\pi}_0 \sum_{i=1}^{n} P_{a_i} \), of normalized reference powers of each generator with \( P_{a_i} = \bar{P}_{a_i} - \bar{P}_z \) where \( \bar{P}_{a_i} \) is the reference power of the power generator of node \( i \) and \( \bar{P}_z \) is a load at node \( i \), and where \( D_i \) is the damping of said power generator at node \( i \),
wherein $A_p$ is the load/generation step occurring in the power supply system and wherein $\mathcal{E}$ is the set of all generator pairs $(i,j)$ that are connected by a power line.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, the admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point of said power supply system is achieved if the following condition is fulfilled:

\[
\left( \sum_{i=1}^{n} M_i \right) \left( \begin{array}{c} \sum_{i,j} a_{ij} \end{array} \right) \geq 2 \left( \sum_{i,j} a_{ij} \right) \arcsin^2 \left( \frac{\|B^T L' \Delta_p\|_\infty}{2 \sqrt{1 - \max \{\|B^T L' \rho\|_\infty, \|B^T L' (\rho + \Delta p)\|_\infty\}} \right)
\]

\[
\leq 2 \|B^T L' \rho\|_\infty - \|B^T L' (p + \Delta p)\|_\infty \arccos \left( \frac{\|B^T L' (p + \Delta p)\|_\infty}{\|B^T L' \rho\|_\infty} \right), \quad \forall \Delta p \in P
\]

wherein $D_i$ is the damping of said power generator at node $i$, $M_i$ is the inertia of said power generator at node $i$, $a_{ij} = b_{ij} - u_{ij}$, with $b_{ij}$ being the susceptance of a power line connecting node $i$ with a neighboring node $j$ of said power supply network, $u_{ij}$, being voltages at node $i$ and its neighboring node $j$ connected via said power line, wherein $B^T$ is the incidence matrix of the network topology of said power supply network, $L = \text{Bdiag}(a_{ij}) B^T$, is a pseudo-inverse of said power supply network's Laplacian matrix $L = \text{Bdiag}(a_{ij}) B^T$. 


wherein $p$ is the vector of $p_i(D,p_{i0}) = p_{i0} - D_i \sum_{i=1}^{N} P_{i0} \sum_{i=1}^{N} D_{i}$ of normalized reference powers of each generator with $P_{i0} = P_{i0} - p_i$, where $p_{i0}$ is the reference power of the power generator of node $i$ and $p_i$ is a load at node $i$, and where $D_i$ is the damping of said power generator at node $i$.

wherein $Ap$ is the load step occurring in said power supply system and $s$ is a set of all expected load steps and $a$ being $\min_s a_s$.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, a ratio between a resistance and a reactance of a power line of said power supply system is close to zero.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, the power supply network is a high voltage power supply network with a voltage level of at least 20kVolts.

In a further possible embodiment of the power supply system according to the first aspect of the present invention, the central control unit adjusts an active power generated by each power generation unit of said power supply system by controlling said power generation unit via a control data network or via power line communication.

According to a further second aspect of the present invention, a method for a load step robust unit commitment of power generation units in a power supply system is provided, wherein an active power generated by each power generation unit of said power supply system is adjusted before occur-
rence of at least one load/generation step within said power supply system to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system to a stable post-load/generation step operating point of said power supply system.

In a possible embodiment of the method according to the second aspect of the present invention, the admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point is such that a power system blackout of said power supply system is avoided.

In a still further possible embodiment of the method according to the second aspect of the present invention, the adjustment of the active power generated by each power generation unit is performed periodically.

In a further possible embodiment of the method according to the present invention, the adjustment of the active power generated by each power generation unit is performed in response to a monitored event within said power supply system.

The invention further provides, according to a third aspect, a control unit of a power supply system wherein said control unit is adapted to adjust an active power of power generation units before occurrence of at least one load/generation step within said power supply system to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system to a stable post-load/generation step operating point of said power supply system.
In a possible embodiment of the control unit according to the third aspect of the present invention, the admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point of said power supply system is achieved if following condition is fulfilled:

\[
\frac{(\sum_{i=1}^{n} M_i)(\sum D_i)\frac{A_D}{2} + 2 \sum_{i,j=1}^{n} a_i) \arcsin \left( \frac{\left\| B^T L^i \right\|_\infty}{2 \sqrt{1 - \eta \max(\left\| B^T L^i (p + \Delta p) \right\|_\infty, \left\| B^T L^i (p + \Delta p) \right\|_\infty)} \right)}{2} \right) \leq 2 \frac{\left\| B^T (p + \Delta p) \right\|_\infty}{\left\| B^T L^i (p + \Delta p) \right\|_\infty} \arccos(\frac{\left\| B^T L^i (p + \Delta p) \right\|_\infty}{\left\| B^T L^i (p) \right\|_\infty}) \right), \quad \forall \text{Ap} \in P
\]

wherein \( D_i \) is the damping of said power generator at node \( i \),
\( M_i \) is the inertia of said power generator at node \( i \),
\( a_i = b_{ij} - a_j \), with
\( b_i \) being the susceptance of a power line connecting node \( i \) with a neighboring node \( j \) of said power supply network,
\( u_i, u_j \) being voltages at node \( i \) and its neighboring node \( j \) connected via said power line,
wherein \( B^i \) is the incidence matrix of the network topology of said power supply network,
\( \hat{\mu} \) is a pseudo-inverse of said power supply network’s Laplacian matrix \( L = B \text{diag}(a_j)B^T \),
wherein \( p \) is the vector of \( p_i(D, p_0) = p_{i0} - D_i \sum_{i=1}^{N} p_{i0} \sum_{i=1}^{N} D_i \) of normalized reference powers of each generator with \( P_{i0} = p_{i0} \cdot p_{d} \), where \( p_{i0} \) is the reference power of the power generator of node \( i \) and \( p_d \) is a load at node \( i \), and where \( \mathcal{D}_i \) is the damping of said power generator at node \( i \).
wherein $\Delta p$ is the load step occurring in said power supply system and $g$ is a set of all expected load steps and $a$ being $\min_y a_y$.

In the following, possible embodiments of different aspects of the present invention are described in more detail with reference to the enclosed figure.

Fig. 1 shows a diagram for illustrating a possible embodiment of a power supply system employing a method and apparatus for a load step robust unit commitment of power generation units according to the present invention.

In the following, different aspects of the present invention are described with reference to Fig. 1. The invention relates to a power supply system PSS comprising a plurality of power generation units PGU connected to each other via power lines of said power supply system. Fig. 1 illustrates an exemplary implementation of a power supply system PSS having a plurality of power generation units PGU connected to each other via power lines. The power supply system PSS shown in Fig. 1 comprises an electrical power supply grid for delivering electricity from electrical suppliers to electrical consumers. In the exemplary implementation shown in Fig. 1, the power supply system comprises a transmission grid TG to which at least one distribution grid DG is connected. The transmission grid TG is adapted to transport electrical power over longer distances. On arrival of a substation, the transported electrical power can be stepped down from a transmission voltage level to a distribution voltage level. There can be power generation units PGU connected to the transmission TG and power generation units PGU connected to the distribution grid.
DG as illustrated in Fig. 1. An example for power generation units connected to the high voltage transmission grid TG are nuclear power plants NPP, hydroelectric power plants HEPP, medium-sized power plants MSPP, industrial power plants IPP, and coal power plants CPP as shown in Fig. 1. Examples for power generation units PGU connected to the distribution grid DG can be city power plants CPP, solar farms SF as well as wind farms WF as shown in Fig. 1. The transmission grid can comprise an extra high voltage transmission grid, for instance in a range of 265 to 275kV connected to a high-voltage power supply grid having a voltage level of H0kV and more. The distribution grid DG can comprise a lower voltage level of for instance 50kV. The distribution grid DG can comprise a city network and rural networks as well as single houses such as farms connected to the distribution grid DG as shown in Fig. 1. Moreover, industrial customers can be connected to the distribution grid.

In the power supply system PSS comprising a plurality of power generation units PGU, at least one control unit CU can be provided which is adapted to adjust an active power generated by each or at least by some power generation units PGU before occurrence of at least one load step and/or generation step within said power supply system PSS to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system PSS to a stable post-load/generation step operating point of said power supply system PSS. The control unit CU can be provided by a central control unit or formed by distributed control entities communicating with each other by peer to peer mechanism.

The power generation units PGU form nodes of a power supply network which comprises power lines connecting the power gen-
eration units and a plurality of loads L. The loads L can be formed by factories connected to the transmission grid TG and/or to the distribution grid DG. Moreover, the loads L can be formed by consumers of the city or rural network connected to the distribution grid DG.

According to a further aspect of the present invention, a method for a load step robust unit commitment of power generation units in a power supply system PSS as shown in Fig. 1 is provided. According to the method, an active power generated by each power generation unit PGU of the power supply system PSS is adjusted before occurrence of at least one load step and/or generation step within said power supply system to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system to a stable post-load/generation step operating point of said power supply system. The method can be performed in a possible embodiment by a program executed by a processor of said control unit CU. The control unit CU performs the task of unit commitment in the distributed power supply system PSS to calculate a unit commitment schedule that minimizes the commitment and dispatch requirements meeting a system load by taking into account various physical constraints of the generating resources, i.e. power generation units PGU, power transmission as well as power supply system reliability requirements. The control unit CU performs an adjustment of the active power generated by the power generation units if a load step or a generation step occurs in the power supply system PSS. A load step can for instance occur when a load L such as a factory or a rural or city network connected to the power supply system PSS is switched on or off. A generation step can occur when a power generation unit PGU such as a wind farm does no longer supply electrical power because
there is no more wind. With the method and apparatus according to the present invention, it is possible to provide an admissible transition from a stable operating point before a load or generation step happens to a stable operating point after occurrence of the load or generation step. This is achieved by adjusting the active power generated by each power generation unit PGU in advance before the load or generation step happens. For instance, the control unit CU can set an operation point for a first power generation unit PGU1 to generate 10 Megawatt of electrical power, a second power generation unit PGU2 can generate 250 Megawatt, a third power generation unit PGU3 can generate power of 300 Megawatt etc. This setting of the operation points of the different power generation units PGUs is performed in advance before a load or generation step \( \Delta p \) occurs in the power supply system PSS and is done in such a way that an admissible transition from the pre-load/generation step operating point to the stable post-load/generation step operating point is achieved. The admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point of the power supply system is achieved if the following condition is satisfied:

\[
\left( \sum_{i=1}^{n} M_i \right) \left( \sum_{i,j=1}^{n} D_{ij} \right) \Delta p + 2 \left( \sum_{i,j=1}^{n} a_{ij} \right) \arcsin^2 \left( \frac{ \left\| (B^T L') \Delta p \right\|_\infty }{ 2 \sqrt{1 - \max \left( \left\| B^T L' \Delta p \right\|_\infty, \left\| B^T (L' + \Delta L') \right\|_\infty \right) } } \right) 
\leq 2d \left[ 1 - \left( B^T L' (p + \Delta p) \right) \arccos \left( \frac{ \left\| B^T L' (p + \Delta p) \right\|_\infty }{ \left\| B^T L' \right\|_\infty } \right) \right], \forall \Delta p \in P
\]

wherein \( \omega_i \) is the damping of said power generator at node \( i \), \( M_i \) is the inertia of said power generator at node \( i \),

\[
a_{ij} = b_{ij} \cdot u_i \cdot u_j
\]

with
by being the susceptance of a power line connecting node \( i \) with a neighboring node \( j \) of said power supply network, \\
u, \( u_j \) being voltages at node \( i \) and its neighboring node \( j \) connected via said power line, \\
wherein \( B^T \) is the incidence matrix of the network topology of said power supply network, \\
\( L^* \) is a pseudo-inverse of said power supply network's Laplacian matrix \( L = B \, \text{diag}(a_j) B^T \), \\
wherein \( p \) is the vector of \( p_i = p_i^D - D_i \frac{\sum_{i=1}^{N} P_{D0}}{\sum_{i=1}^{N} D_i} \) of normalized reference powers of each generator with \( p_{io} = \tilde{p}_{D0} - p_{il} \) where \( \tilde{p}_{D0} \) is the reference power of the power generator of node \( i \) and \( p_a \) is a load at node \( i \), and where \( D_i \) is the damping of said power generator at node \( i \), \\
wherein \( A_p \) is the load step occurring in said power supply system and \( a \) is a set of all expected load steps and \( a \) being \( \min_{\_a} \).

A general power grid or power supply system PSS can comprise synchronous generators and inverters as sources, asynchronous machines, rectifiers and passive loads as sinks, and purely inductive lines. A simplified model for frequency stability analysis of this power supply grid is given by

\[
M_i \dot{\theta}_i + D_i \ddot{\theta}_i = \tilde{p}_{D0} - p_{il} - \sum_{j \in N_i} b_{ij} u_i u_j \sin(\tilde{\theta}_i - \tilde{\theta}_j)
\]

(1)

for \( i \in N = \{1, \ldots, n\} \), where \( \tilde{p}_{D0}, D_i > 0, M_i \geq 0 \) are the reference power as well as the damping and the inertia of the synchronous generators at node \( i \). The voltage phase angle \( \tilde{\theta}_i \) of generator
is described in a coordinate frame rotating with the nominal grid frequency, e.g. \( f_0 = 50 \text{Hz} \) (\( \omega_0 = \frac{2\pi f_0}{60} \)). The set of neighbors of node \( i \) is \( N_i \). Moreover, \( p_i \) is the load at bus \( i \), \( b_i \) is the susceptance of the power line, and \( u_i, u_j \) are the rounded mean square (rms) values of the voltage at nodes \( i, j \). For ease of notation, one can define \( p_{i0} = p_i - p_{iL} \) and \( a_i = b_i u_i u_j \).

This system can be transformed in a rotating frame where the rotation frequency is the steady state frequency \( \omega_0 + \omega_x \).

Thereby, \( \omega_x \) denotes the steady state frequency deviation from the nominal frequency \( f_0 \). In steady state, the sum of in-flowing and out-flowing power of the power supply grid is zero, i.e.

\[
\sum_{i=1}^{N} \sum_{j \in N_i} b_i u_i u_j \sin (\tilde{\theta}_i - \tilde{\theta}_j) = 0, \tag{2}
\]

where \( \tilde{\theta}_i - \tilde{\theta}_j \) denotes the constant steady state deviation between \( \tilde{\theta}_i \) and \( \tilde{\theta}_j \).

Therefore, one has

\[
\sum_{i=1}^{N} D_i \omega_x = \sum_{i=1}^{N} p_{i0}, \tag{3}
\]

from where one gets

\[
\omega_x = \frac{\sum_{i=1}^{N} p_{i0}}{\sum_{i=1}^{N} D_i}, \tag{4}
\]
The transformation in a rotating frame is
\[ \theta_i = \tilde{\theta}_i - \omega_i, \quad \dot{\theta}_i = \tilde{\theta}_i - \omega_n, \] and \( e_i = \tilde{\theta}_i \) and one obtains:

\[ M_i \dot{\tilde{\theta}}_i + D_i \tilde{\theta}_i = P_i \varphi \cdot p_0 - \sum_{j=N_i} a_i \sin(\tilde{\theta}_i - \theta_j), \] \hspace{1cm} (5)

where

\[ p_i(D,p_0) = p_{i0} - D_i \sum_{j=1}^{N} p_j \delta_{ij}, \] \hspace{1cm} (6)

depends on the dampings \( D = \text{vec}(D_i) \) and reference powers \( p_0 = \text{vec}(p_{i0}) \). For ease of notation, one can write \( p_i \) when the argument \( D,p_0 \) is not important. Note that \( \sum_{i=1}^{N} p_i(D,p_0) = 0 \) for all \( D \) and \( p_0 \).

The steady state is reached when all the generators rotate with the same frequency \( \omega_n + \omega_n \), retaining constant phase differences \( \theta_i^{(0)} = \theta_i^e - \theta_j^e, \forall i, j \in \mathcal{E} \), i.e., \( \lim_{t \to \infty} \theta_i(t) - \theta_j(t) = \theta_{i}^{(0)}, \forall i \in \{1, \ldots, n\} \) if the power supply system PSS is asymptotically stable. The superscript is used to distinguish this equilibrium from another one defined later on. The equilibrium phase differences \( \theta_i^{e(m)}, i \in \mathcal{N} \) satisfy

\[ 0 = p_i(D,p_0) - \sum_{j=\mathcal{N}_i} a_{ij} \sin(\theta_j^{(m)}), \] \hspace{1cm} (7)

for all \( \mathcal{N} \). This can be written in compact form as

\[ p = B \text{ diag}_{\alpha} \text{ sm}(B^T \theta^{(m)}), \] \hspace{1cm} (8)
where $B^T$ is the incidence matrix of the network topology, $p = \text{vec}(p_i)$, and $\theta^{(1)} = \text{vec}(\theta^{m})$, respectively. A locally asymptotically stable equilibrium $\theta^{(1)}$ exists if $\|B^T L^p\|_\infty < 1$ and the equilibrium satisfies

$$\max_{(i,j) \in E} |\theta^{(1)}_{ij}| \leq \arcsin(\sqrt{\|L^T L^p\|_\infty}).$$

(9)

Thereby, $L^\prime$ is the pseudo-inverse of the network's Laplacian matrix $L = B \text{diag}(a_j) B^T$. Note that phase differences of power generators that are not connected need not be bounded by $\arcsin(\|B^T L^p\|_\infty)$, i.e. $\|\theta^{(1)}_{ij}\| \leq \arcsin(\sqrt{\|L^T L^p\|_\infty})$ is not required for any $(i,j) \not\in E$.

Now, when considering stepwise load and generation variations, these variations can be due to load changes or power drops in renewable energy sources, e.g. photovoltaic or wind power. These variations result in changes in $p_\pm$ or $\tilde{p}_j$, respectively, in Equation (1), or in $p_i$ in equation (5), see (6). Hence, a load/generation step is expressed as $p + \Delta p$.

The equilibrium phase differences is denoted before and after the load step as $\theta^{(1)}_{ij}$ and $\theta^{(2)}_{ij}$, respectively. The distance between the equilibria $\Delta \theta^{(2)}_{ij} = \theta^{(2)}_{ij} - \theta^{(1)}_{ij}$ is related to the load step $\Delta p$. A locally asymptotically stable equilibrium exists after the load step if $\|B^T L^\dagger (p + \Delta p)\|_\infty \leq 1$ and the equilibrium satisfies
Moreover, one can derive the following bound on the distance between the equilibria

$$\max_{(i,j) \in \delta} |\theta_{ij}^{(2)}| \leq \arcsin\left( \frac{\|B^T L'(p + \Delta p)\|_\infty}{\|B^T L' A p\|_\infty} \right). \quad (10)$$

Moreover, one can derive the following bound on the distance between the equilibria

$$\max_{(i,j) \in \delta} |\Theta_{ij}| \leq 2 \arcsin\left( \frac{\|B^T L' A p\|_\infty}{2 \sqrt{1 - \max \{ \|B^T L' p\|_\infty^2, \|B^T L'(p + \Delta p)\|_\infty^2 \} }} \right). \quad (11)$$

The fulfilment of this condition can be calculated by a processor or calculation unit of the control unit CU.

In a preferred embodiment of the power supply system PSS according to the present invention, a ratio between a reactance and an impedance of a power line within said power supply system PSS is close to zero. Accordingly, in a possible embodiment, the power supply network is a high voltage power supply network having a voltage level of at least 20kVolts.

In a possible embodiment, the control unit CU can be formed by a central control unit which adjusts an active power generated by each or at least most power generation units PGU by controlling said power generation unit via a control data network or via power line communication PLC. In a possible embodiment, the adjusting of the active power generated by each power generation unit PGU is performed by the control unit CU periodically, for instance every day or every hour.

In an alternative embodiment, the adjustment of the active power generated by the power generation units PGU is performed in response to a monitored event within said power supply system PSS.
Now, conditions for load/generations step robustness are provided. The case that the system (1) is in a steady state before the load generation step is considered, i.e.

\[ \tilde{\omega}_4(0) = \omega_{i4}^{(1)}, \quad \tilde{\varphi}(0) = \varphi, \quad \tilde{\psi}(0) = 0. \]  

(12)

The load/generation step occurs at time \( t = 0 \), i.e. (12) is the initial condition of the dynamical system (1) with \( p_{i0} \) replaced by \( p_{i0} + A p_{i0} \). The following conditions (13) ensure that, after the load/generation step \( A p \), the dynamical system (1) has an asymptotically stable equilibrium and that the initial condition (12) lies inside the region of attraction of this equilibrium. In other words, the power supply system PSS eventually shows a transition from the equilibrium before the load step to the equilibrium after the load step.

\[ \| B^T L_p \|_\infty < \gamma, \]
\[ \| B^T L_p(p + \Delta p) \|_\infty < \gamma, \]
\[ (\sum_{i=1}^{n} M_i) (\sum_{i,j} A_{ij})^2 + 2(\sum_{i,j} a_{ij}) \arcsin^2(\frac{\| B^T L_p \|_\infty}{2}) \leq 2d(1 - \| B^T L_p(p + \Delta p) \|_\infty) \arccos(\| B^T L_p(p + \Delta p) \|_\infty / \gamma^2), \]  

(13c, 13b)

where \( a = \min_j a_{ij} \). Conditions (13c) and (13b) ensure that the equilibria before and after the load/generation step are locally asymptotically stable and that the steady state phase differences before and after the load step are bounded by \( \gamma < \gamma \). Condition (13c) guarantees that the power supply system PSS evolves from one equilibrium to the other.
Condition (13) can be used to calculate a load/generation step robust unit commitment. Unit commitment usually optimizes the power generation with respect to the generation effort. Therefore, a cost or effort function $C(p)$ can be defined as

$$C(p) = \sum_i c_i(\tilde{p}_{i0}),$$  \hspace{1cm} (14)$$

where $c_i(\tilde{p}_{i0})$ is the effort of generating power $\tilde{p}_{i0}$ at generator $i$. In order to guarantee power production according to the power consumption, an inequality constraint can be added for some $\varepsilon > 0$. Note that the mismatch between generation and consumption, i.e. $\sum_i \tilde{p}_{i0} + p_u$, directly influences the steady state frequency deviation $\omega_u$. Therefore, $\varepsilon$ should be small.

Using condition (13), one can extend this to a load/generation step robust unit commitment:

$$\min_{\tilde{p}_{i0}} \sum_i c_i(\tilde{p}_{i0})$$  \hspace{1cm} (16a)$$

$$\text{s.t.} \left| \sum_i \tilde{p}_{i0} + p_u \right| < \varepsilon$$  \hspace{1cm} (16b)$$

$$\|B^T L p\|_\infty < \gamma,$$  \hspace{1cm} (16c)$$

$$\|B^T L (p + \Delta p)\|_\infty < \gamma, \ \forall \Delta p \in P$$  \hspace{1cm} (16d)$$

$$\left(\sum_{i=1}^n M_i \right) \left(\sum_{i,j=1}^n \frac{\Delta p_{ij}}{D_i} \right)^2 + 2 \left(\sum_{i,j=1}^n a_{ij} \right) \arcsin^2 \left(\frac{\|B^T L \Delta p\|_\infty}{2 \sqrt{1 - \max \{\|B^T L p\|_\infty, \|B^T L (p + \Delta p)\|_\infty\}^2}}\right)$$
where (16a) achieves the minimization of the generation effort, (16b) guarantees the matching between generated and consumed power with design parameter $\varepsilon$, (16c) and (16d) guarantee bounded steady state phase differences before and after the load/generation step with the design parameter $\gamma < 1$, and (16e) ensures a stable and reliable transient between the equilibria before and after the load/generation step $\Delta p$ for all possible load/generation steps in a predefined set $P$, which is again an optimization parameter.

The optimization problem (16) is quite complex because of the nonlinear constraint (16e). This constraint can be simplified by using (16c) and (16d) leading to the following more conservative constraint:

$$\leq 2a\sqrt{1 - \|B^T L'(p + \Delta p)\|^2_x} + \frac{2 \|B^T L'(p + \Delta p)\|_x \arccos(\|B^T L'(p + \Delta p)\|_x)}{2\sqrt{1 - \gamma^2}}, \forall \Delta p \in P$$

(16e)

because $\sqrt{1 - \gamma^2} - \gamma \arccos(\gamma)$ is decreasing for $\gamma \in (0,1)$.

The optimization problem (16) with (17) can be solved as follows for given $c_i, p_x, s, B, a_j, M_j, D_i$:

In a first step $S_1$ a set of admissible load/generation steps $P$ can be defined or loaded.
In a second step $S_2$
\[
\begin{align*}
\min_{\gamma \in (0,1)} & \quad \gamma \\
\text{s.t.} & \quad (17) \text{ holds}
\end{align*}
\]

is calculated by a processor.

5

The calculated solution is \(\gamma^* > 0\). If no solution exists, it is looped back to the first step SI and a smaller set of admissible load/generation steps \(P\) is defined.

10 In a third step S3

\[
\begin{align*}
\min_{\vec{p}_0} & \quad \sum_i c_i(\vec{p}_0) \\
\text{s.t.} & \quad \left| \sum_i \vec{p}_i + p_d \right| < \varepsilon \\
& \quad \left\| B^T L' \rho \right\|_\infty < \gamma^*, \\
& \quad \left\| B' L' (p + Ap) \right\|_\infty < \gamma^*, \ \forall \rho \in P
\end{align*}
\]

is calculated by the calculation unit or processor.

The solution is \(\vec{P}_0^*\). If no solution exists, it is looped back to step SI and a smaller set of admissible load/generation steps \(P\) is loaded.

If the admissible load/generation step set \(P\) is a polytope, then the infinitely many constraints in (17) and (19d) can be replaced by the corresponding constraints of the corner points of the polytope. In this case, (18) can be solved efficiently using convex optimization techniques. If, moreover, the effort functions \(c_i\) are convex, then the optimization problem (19) can also be solved efficiently.
using a convex optimization performed by a calculation entity of the control unit.
Claims

1. A power supply system (PSS) comprising a plurality of power generation units (PGU) connected to each other via power lines and comprising at least one control unit (CU) adapted to adjust an active power generated by each power generation unit (PGU) before occurrence of at least one load step and/or generation step within said power supply system (PSS) to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system (PSS) to a stable post-load/generation step operating point of said power supply system (PSS).

2. The power supply system according to claim 1, wherein said power generation units (PGU) form nodes of a power supply network which comprises power lines connecting the power generation units (PGU) and a plurality of loads.

3. The power supply system according to claim 1, wherein the power generation unit (PGU) of a node comprises a synchronous generator having an associated reference power, damping and inertia.

4. The power supply system according to claim 1, wherein for each node of said power supply network (PSS), the phase dynamics of each generator follow the differential equations:

\[
M_i \ddot{\theta}_i + D_i \dot{\theta}_i = \bar{P}_{\text{ref},i} - P_{IL} - \sum_{j \in N_i} b_{ij} U_j \sin(\theta_i - \phi_j),
\]
wherein $\bar{p}_{i0}$ is the reference power of the power generator of said node $i$,

$p_i$ is a load at node $i$,

$\sigma_i$ is the damping of said power generator at node $i$,

$M_i$ is the inertia of said power generator at node $i$,

$b$ is the susceptance of a power line connecting said node $i$ with a neighboring node $j$ of said power supply network,

$u_{i,j}$ are voltages at node $i$ and its neighboring node $j$ connected to each other via said power line,

$\theta', \Theta$ are phase angles at node $i$ and its neighboring node $j$ connected to each other via said power line.

5. The power supply system according to claim 4, wherein a bound of the maximum distance of $(\theta_{ij}^\infty)$ between the phase difference $(\theta_{ij}^{x(1)})$ at the stable pre-load step operating point before occurrence of the load step ($Ap$) and the phase difference $(\theta_{ij}^{x(2)})$ at the stable post-load step operating point after occurrence of the load step ($Ap$) is given by:

$$\max_{(i,j)\in E} |\Delta \Theta_{ij}| \leq 2 \arcsin\left(\frac{\|B^T L' Ap\|_F}{2\sqrt{1-\max\{\|5^TL' p\|_F^2, \|B^T L'(p + Ap \hat{W})\|_F^2\}}}\right)$$

wherein $\Delta \Theta_{ij}$ is the difference between $\theta_{ij}^{x(1)}$ and $\theta_{ij}^{x(2)}$,

wherein $B^T$ is an incidence matrix of the network topology of said power supply system,

$\hat{\Omega}$ is a pseudo-inverse of the power supply network's Laplacian matrix $L = B \text{diag}(\lambda^i) B^T$, 

$$L = B \text{diag}(\lambda^i) B^T$$
wherein \( \mathbf{a} = \mathbf{b} \times \mathbf{u} \times \mathbf{v} \),

wherein \( \mathbf{p} \) is the vector of \( \mathbf{p}(D, \mathbf{p}_0) = \mathbf{p}_0 - D - \frac{\sum_{i=1}^{N} p_{i0}}{\sum_{i=1}^{N} D_i} \) of normalized reference powers of each generator with \( p_{i0} = \tilde{p}_{i0} - p_d \)

where \( \tilde{p}_{i0} \) is the reference power of the power generator of node \( i \) and \( \mathbf{p}_0 \) is a load at node \( i \), and where \( D_i \) is the damping of said power generator at node \( i \),

wherein \( \mathbf{A} \mathbf{p} \) is the load step occurring in said power supply system and

wherein \( \mathcal{S} \) is the set of all generator pairs \( (i,j) \) that are connected by a power line.

6. The power supply system according to claim 5, wherein the admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point of said power supply system is achieved if the following condition is satisfied:

\[
\left( \sum_{i=1}^{N} M_i \right) \left( \sum_{i=1}^{N} \frac{\Delta p_i}{D_i} \right)^2 + 2 \left( \sum_{i,j=1}^{N} a_{ij} \right) \arcsin^2 \left( \frac{\left\| B^T L' \Delta p \right\|_2}{2 \sqrt{1 - \max \left\{ \left\| B^T L' p \right\|_2, \left\| B^T L' (p + \Delta p) \right\|_2 \right\}} \right) \\
\leq 2 \Delta \left( 1 - \left\| B^T L' (p + \Delta p) \right\|_2 \right)^2 - \left\| B^T L' (p + \Delta p) \right\|_2 \arccos \left( \frac{\left\| 5' V (p + \Delta p) \right\|_2}{\left\| B^T L' (p + \Delta p) \right\|_2} \right) \right) \forall \mathbf{A} \mathbf{p} \in \mathbf{p}
\]

wherein \( D_i \) is the damping of said power generator at node \( i \),

\( M_i \) is the inertia of said power generator at node \( i \),

\( \mathbf{a} = \mathbf{b} \times \mathbf{u} \times \mathbf{v} \) with

\( \mathbf{b} \) being the susceptance of a power line connecting node \( i \) with a neighboring node \( j \) of said power supply system,
\( u_i \) \( u_j \) being voltages at node \( i \) and its neighboring node \( j \) connected via said power line,

wherein \( B^T \) is the incidence matrix of the network topology of said power supply system,

\( \hat{u} \) is a pseudo-inverse of said power supply network's Laplacian matrix \( L = B \text{diag}(a_y)B^T \),

wherein \( p \) is the vector \( p_i(D, p_0) = p_0 - D \sum_{i=1}^{N} p_{10} / \sum_{i=1}^{N} D_i \) of normalized reference powers of each generator with

\( p_{10} = \tilde{p}_{10} - p_d \) where \( \tilde{p}_{10} \) is the reference power of the power generator of node \( i \) and \( p_i \) is a load at node \( i \), and

where \( D_i \) is the damping of said power generator at node \( i \),

wherein \( a_p \) is the load step occurring in said power supply system and \( \mathcal{D} \) is a set of all expected load steps and \( a \) being \( \min\_{\mathcal{D}} a \).

7. The power supply system according to claim 1, wherein a ratio between a resistance and a reactance of a power line within said power supply system (PSS) is close to zero.

8. The power supply system according to claim 7, wherein said power supply system (PSS) comprises a high voltage power supply network with a voltage level of at least 20kVolts.

9. The power supply system according to claim 1, wherein said control unit (CU) adjusts an active power generated by each power generation unit (PGU) by controlling said
power generation unit (PGU) via a control data network or via power line communication.

10. A method for a load step robust unit commitment of power generation units (PGU) in a power supply system (PSS), wherein an active power generated by each power generation unit (PGU) of said power supply system (PSS) is adjusted before occurrence of at least one load step and/or generation step within said power supply system (PSS) to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system (PSS) to a stable post-load/generation step operating point of said power supply system (PSS).

11. The method according to claim 10, wherein the admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point is such that it avoids a power system blackout of said power supply system (PSS).

12. The method according to claim 10, wherein the adjusting of the active power generated by each power generation unit (PGU) is performed periodically or in response to a monitored event within said power supply system (PSS).

13. A control unit of a power supply system, said control unit (CU) being adapted to adjust an active power of power generation units (PGU) before occurrence of at least one load step and/or generation step within said power supply system (PSS) to provide an admissible transition from a stable pre-load/generation step operating point of said power supply system (PSS) to a stable
post-load/generation step operating point of said power supply system (PSS).

14. The control unit according to claim 13, wherein the admissible transition from the stable pre-load/generation step operating point to the stable post-load/generation step operating point of said power supply system (PSS) is achieved if the following condition is satisfied:

\[
\left(\sum_{i=1}^{n} M_i \left(\sum_{j=1}^{n} \frac{\Delta p_j}{D_i}\right)^2 + 2 \left(\sum_{i,j=1}^{n} a_{ij}\right) \arcsin^2 \left(\frac{\|B^T L'(p + \Delta p)\|_2}{2\sqrt{1 - \|B^T L'(p + \Delta p)\|_2^2}}\right)\right) \\
\leq 2 \|B^T L' (p + \Delta p)\|_2^2 - \|B^T L' (p + \Delta p)\|_2 \arccos \left(\|B^T L'(p + \Delta p)\|_2\right), \forall \Delta p \in P
\]

wherein \(D_i\) is the damping of said power generator at node \(i\),

\(M_i\) is the inertia of said power generator at node \(i\),

\(a_{ij} = b_{ij} \cdot u_i \cdot u_j\) with

\(b_{ij}\) being the susceptance of a power line connecting node \(i\) with a neighboring node \(j\) of said power supply network,

\(u_i, u_j\) being voltages at node \(i\) and its neighboring node \(j\) connected via said power line,

wherein \(B^T\) is the incidence matrix of the network topology of said power supply system,

\(L^T\) is a pseudo-inverse of said power supply network's Laplacian matrix \(L = B \text{diag}(a_i) B^T\),

wherein \(p\) is the vector of \(p_{i}(D, p_{0}) = p_{i0} - D_i \sum_{j=1}^{n} p_{j0} \sum_{i=1}^{N} D_i\) of normalized reference powers of each generator with \(p_{j0} = \hat{p}_{j0} \cdot p_{d}\)

where \(\hat{p}_{j0}\) is the reference power of the power generator of
node $i$ and $p_i$ is a load at node $i$, and where $D_i$ is the damping of said power generator at node $i$, wherein $A_p$ is the load step occurring in said power supply system and $\mathcal{L}$ is a set of all expected load steps and $a$ being $\min_{\mathcal{L}} a_{y}$. 
INTERNATIONAL SEARCH REPORT

PCT/EP2013/068865

A. CLASSIFICATION OF SUBJECT MATTER

INV. H02J3/24
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H02J G05B

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
EPO-Internal , WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

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Date of the actual completion of the international search
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Name and mailing address of the ISA/

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