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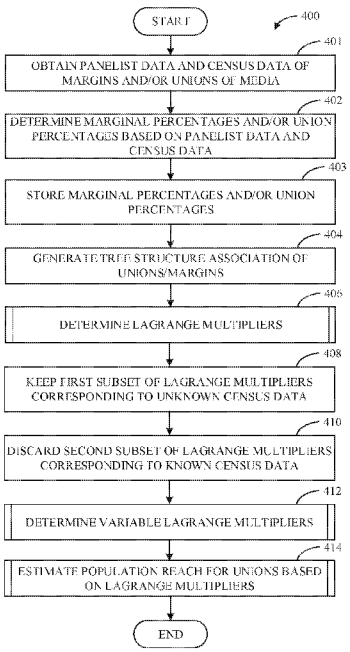


FIG. 4

(57) Abstract: Example methods, apparatus, systems, and articles of manufacture are disclosed to estimate population reach for different unions based on marginal ratings. An example apparatus includes an association controller to generate a tree association corresponding to a union of a first and second margin of time; and one or more commercial solvers to perform parallel computations on a processor to determine first multipliers corresponding to a total number of panelists exposed to media at the first margin, the second margin, and the union by solving first equations corresponding to the tree association; discard a first multiplier and a second multiplier of the first multipliers corresponding to the first margin and the second margin; perform parallel computation on the processor to determine second multipliers corresponding to a total audience exposed to the media at the first margin and the second margin by solving second equations corresponding to the tree association, the second system of equations including a subset of the first multipliers; and determine an estimate for a population reach of the union based on the second multipliers.

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METHODS AND APPARATUS TO ESTIMATE POPULATION REACH FROM DIFFERENT MARGINAL RATING UNIONS

RELATED APPLICATIONS

[0001] Priority is claimed to U.S. Patent Application No. 16/355,393, filed on March 15, 2019, which is hereby incorporated herein by reference in its entirety.

FIELD OF THE DISCLOSURE

[0002] This disclosure relates generally to media audience measurement, and, more particularly, to methods and apparatus to estimate population reach from different marginal rating unions.

BACKGROUND

[0003] Determining a size and demographics of an audience of a media presentation helps media providers and distributors schedule programming and determine a price for advertising presented during the programming. In addition, accurate estimates of audience demographics enable advertisers to target advertisements to certain types and sizes of audiences. To collect these demographics, an audience measurement entity enlists a group of media consumers (often called panelists) to cooperate in an audience measurement study (often called a panel) for a predefined length of time. In some examples, the audience measurement entity obtains (e.g., directly, or indirectly from a media service provider) return path data (e.g., census data representative of a population of users) from media presentation devices (e.g., set-top boxes) that identifies tuning data from the media presentation device. In such examples, because the return path data may not be associated with a known panelist, the audience measurement entity models and/or assigns viewers to represent the return path data. Additionally, the media consumption

habits and demographic data associated with the enlisted media consumers are collected and used to statistically determine the size and demographics of the entire audience of the media presentation. In some examples, this collected data (e.g., data collected via measurement devices) may be supplemented with survey information, for example, recorded manually by the presentation audience members.

BRIEF DESCRIPTION OF THE DRAWINGS

[0004] FIG. 1 is a block diagram of an example environment in which return path data and meter data are collected from media presentation locations and are analyzed by an example audience measurement entity to estimate media reach for a population for different unions of audience marginal ratings data.

[0005] FIGS. 2A-C illustrate example panelist data and census data and corresponding tree structure to represent such data.

[0006] FIG. 3 is a block diagram of an example implementation of the example union population reach determiner of FIG. 1.

[0007] FIGS. 4-7 are flowcharts illustrating example machine readable instructions that may be executed to implement the example union population reach determiner of FIGS. 1 and/or 3.

[0008] FIG. 8 is a block diagram of an example processing system structured to execute the example machine readable instructions of FIGS. 4-7 to implement the example union population reach determiner of FIGS. 1 and/or 3.

[0009] In general, the same reference numbers will be used throughout the drawing(s) and accompanying written description to refer to the same or like parts.

[0010] Descriptors "first," "second," "third," etc. are used herein when identifying multiple elements or components which may be referred to separately. Unless otherwise specified or understood based on their context of use, such descriptors are not intended to impute any meaning of priority, physical order or arrangement in a list, or ordering in time but are merely used as labels for referring to multiple elements or components

separately for ease of understanding the disclosed examples. In some examples, the descriptor "first" may be used to refer to an element in the detailed description, while the same element may be referred to in a claim with a different descriptor such as "second" or "third." In such instances, it should be understood that such descriptors are used merely for ease of referencing multiple elements or components.

DETAILED DESCRIPTION

[0011] Audience measurement entities seek to understand the composition and size of audiences of media, such as television programming. Such information allows audience measurement entity researchers to, for example, report advertising delivery and/or targeting statistics to advertisers that target their media (e.g., advertisements) to particular audiences. Additionally, such information helps to establish advertising prices commensurate with audience exposure and demographic makeup (referred to herein collectively as “audience configuration”). One way to gather media presentation information is to gather the media presentation information from media output devices (e.g., gathering television presentation data from a set-top box (STB) connected to a television). As used herein, media presentation includes media output by a media device regardless of whether or not an audience member is present (e.g., media output by a media output device at which no audience is present, media exposure to an audience member(s), etc.).

[0012] A media presentation device (e.g., STB) provided by a service provider (e.g., a cable television service provider, a satellite television service provider, an over-the-top (OTT) service provider, a music service provider, a movie service provider, a streaming media provider, etc.) or purchased by a consumer may contain processing capabilities to monitor, store, and transmit tuning data (e.g., which television channels are tuned by the media presentation device at a particular time) back to the service provider, which can then aggregate and provide such return path data to an audience measurement entity (e.g., The Nielsen Company (US), LLC.) to analyze media presentation activity. Data transmitted from a media presentation device back to the service provider is referred to

herein as return path data which may include census data. Return path data includes tuning data. Tuning data is based on data received from the media presentation device while the media presentation device is on (e.g., powered on, switched on, and/or tuned to a media channel, streaming, etc.). Although return path data includes tuning data, return path data may not include data related to the user viewing the media corresponding to the media presentation device. Accordingly, return path data may not be able to be associated with specific viewers, demographics, locations, etc. However, census data may be derived or extracted from return path data. Census data is indicative of the total percentage of a population of users (e.g., based on the return path data) that was exposed to media at a particular margin. For example, if 20% of a population was exposed to a first margin (e.g., a first 15 minute segment) of a television show, the census data may be indicative the 20% exposure.

[0013] To determine aspects of media presentation data (e.g., which household member is currently consuming a particular media and the demographics of that household member), market researchers may perform audience measurement by enlisting a subset of the media consumers as panelists. Panelists or monitored panelists are audience members (e.g., household members, users, panelists, etc.) enlisted to be monitored, who divulge and/or otherwise share their media activity and/or demographic data to facilitate a market research study. An audience measurement entity typically monitors media presentation activity (e.g., viewing, listening, etc.) of the monitored panelists via audience measurement system(s), such as a metering device(s) and/or a local people meter (LPM). Audience measurement typically includes determining the identity of the media being presented on a media output device (e.g., a television, a radio, a computer, etc.), determining data related to the media (e.g., presentation duration data, timestamps, channel data, etc.), determining demographic information of an audience, and/or determining which members of a household are associated with (e.g., have been exposed to) a media presentation. For example, an LPM in communication with an audience measurement entity communicates audience measurement (e.g., metering) data to the audience measurement entity. As used herein,

the phrase "in communication," including variances thereof, encompasses direct communication and/or indirect communication through one or more intermediary components and does not require direct physical (e.g., wired) communication and/or constant communication, but rather additionally includes selective communication at periodic or aperiodic intervals, as well as one-time events.

[0014] In some examples, metering data (e.g., including media presentation data) collected by an LPM or other meter is stored in a memory and transmitted via a network, such as the Internet, to a datastore managed by the audience measurement entity. Typically, such metering data is combined with additional metering data collected from a group of LPMs monitoring a group of panelist households. The metering data may include, but are not limited to, a number of minutes a household media presentation device was tuned to a particular channel, a number of minutes a household media presentation device was used (e.g., consumed) by a household panelist member and/or a visitor (e.g., a presentation session), demographics of the audience (which may be statistically projected based on the panelist data), information indicative of when the media presentation device is on or off, and/or information indicative of interactions with the media presentation device (e.g., channel changes, station changes, volume changes, etc.) , etc. As used herein, a channel may be a tuned frequency, selected stream, an address for media (e.g., a network address), and/or any other identifier for a source and/or carrier of media.

[0015] Examples disclosed herein receive the marginal media exposure data (e.g., different episodes of a television series, different quarter hour time slots of a television program, or a radio program, etc.) for different unions of marginal data and/or smaller unions and estimates a total population reach (e.g., a total number of deduplicated users that were exposed to media) across all of the different unions. As used herein, a margin or marginal is a subpart of media. For example, if the media corresponds to advertisement, the margins may be different websites that include the advertisement. In another example, if the media corresponds to a one-hour program, the margins may be 4 15-minute increments of the one-hour program. As used herein, a union can be made up

of smaller unions (e.g., a union of smaller unions of marginals, such as a union of smaller unions of time-periods) and/or individual marginals (e.g., time-periods, such as quarter hours). For example, a first union may include a first quarter hour marginal and a second subsequent quarter hour marginal, a second union may include a third, fourth, and fifth quarter hour marginal, and a third union may include the first union and the second union. As used herein, child unions or children are the marginal time intervals and/or smaller unions that make up a larger union and a parent union or a parent is a larger union that includes the child union(s) and/or children. Using the above example, the children of the first union include the first quarter hour marginal and the second quarter hour marginal and the parent of the first union is the third union.

[0016] In some examples, the audience measurement entity processes the collected and/or aggregated metering data from panelist meters and obtains (e.g., from one or more service provider) return path data for devices where a panel is not maintained. Return path data may include, for example, a total number of or a percentage of unique users (e.g., deduplicated users) from a universe of users that was exposed to media within different margins (e.g., 15 minute increments) of time. However, return path data may be missing a total number of or a percentage of unique users from a universe of users that was exposed to the media within two or more margins of time (e.g., unions). Examples disclosed herein leverage panelist data to be able to estimate unique audiences of universe (e.g., herein also referred to as population) and/or reach(es) across different unions of time. As used herein, reach is a cumulative percentage or total of a population that has been counted as a viewer of the media at least once during a specified time interval (e.g., hourly, daily, weekly, monthly, etc.).

[0017] Some traditional techniques of determining the total population reach for a union of marginal media ratings data include numerical calculations that enumerate all combinations in which someone can watch a program based on the individual marginal media ratings of making up the union. The number of probabilities to be solved by such traditional techniques is 2^n number of probabilities, where n is the number of marginal probabilities provided in the marginal ratings data (e.g., the ratings for the different

possible instances of media exposure) of each union. For example, if reach is to be calculated across 4 quarter hours (e.g., for an hour-long media exposure), which corresponds to four possible marginal ratings, the total number of probabilities to be solved using such a traditional technique is 16 (e.g., 2^4). If the reach is to be calculated across 96 quarter hours (e.g., a day-long media exposure), the total number of probabilities to be solved using such a traditional technique is $8*10^{28}$ (e.g., 2^{96}), thereby exceeding the memory limit and/or processing power of any existing computer. Examples disclosed herein alleviate such memory/processing resource problems associated with such a traditional technique by calculating the solution using the disclosed analytical process.

[0018] To estimate the total population reach of a union (e.g., population union reach), examples disclosed herein develop a tree graph association or tree association for the margins and union(s). The tree graph association corresponds to the structure of the margins and/or unions where each margin and each union corresponds to a node. Examples disclosed herein tag each node as a descendant (e.g., child, grandchild, etc.) and/or ancestor (e.g., parent, grandparent, etc.) depending on the structure of the unions corresponding to stored panel data. For example, if panel data includes a unique audience number or percentage corresponding to three margins (e.g., A, B, and C) and the panel data further includes a unique audience number or percentage corresponding to a first union (e.g., AB) and a second union (e.g., ABC), examples disclosed herein may tag (1) nodes A and B as having parent AB and grandparent ABC, (2) node C as having parent ABC, (3) node AB as having children A and B and parent ABC, and (4) node ABC as having children AB and C and grandchildren A and B. Additionally, examples disclosed herein may tag the margin nodes (e.g., A, B, and C) as leaves and tag ABCD as a root. As used herein, a leaf is a node that does not have children and a root is a node that has no parents.

[0019] As described above, the census data (e.g., corresponding to the population or total audience) may include unique audience data related to particular margins, but may not include unique audience data for unions of the margins. Accordingly, once the tree

structure association is complete, examples disclosed herein utilize the panel data and corresponding tree structure association to solve for unique audience(s) (e.g., reach(es)) of unions of the population based on the census data. The unique audience of a particular union corresponding to a node of the tree structure association can be determined by taking the difference between the total audience and the total audience of the union that was removed from the tree structure association (e.g., the statistical rule of subtraction), as shown the below-Equation 1. Examples disclosed herein leverage Equation 1 to determine unique audiences for a population union.

[0020] AUD(X) = AUD(T)-A(T\X) (Equation 1)

[0021] In Equation 1 above, AUD(X) represents the unique audience of union or margin X (e.g., the audience which belonged to at least one of the margins corresponding to union X), AUD(T) represents the total audience, and AUD(T\X) represents the total audience excluding the audience members of union X. In some examples, the total audience AUD(T) may be known (e.g., for panelist data). The total audience AUD(T) corresponds to the below-Equation 2.

[0022] $AUD(T) = z_0 z_N s_N$ (Equation 2)

[0023] In Equation 2 above, z_0 corresponds to a normalize constraint, N corresponds to the node of the total audience, $z_i = e^{\lambda_i}$, where λ is an unknown Lagrange multiplier (e.g., which can be solved for), and s_N is defined recursively by node height (e.g., margins being nodes at the lowest height, unions of margins being at the next height, etc.) based on the tree association using the Equation 3, as shown below.

[0024] $$s_i = \begin{cases} 1, & if\ node\ i\ is\ a\ leaf \\ \prod_{j \in children(i)} (1 + s_j z_j) - 1, & if\ node\ i\ has\ children \end{cases}$$ (Equation 3)

[0025] As used herein, node height refers to the level or hierarchy level of the tree structure. For example, all the leaf nodes (e.g., margins) correspond to the lowest height and the root node (e.g., the union corresponding to all leaf nodes of a tree association structure) corresponds to the highest height. As the calculations of s_i depend on its children, if any, examples disclosed herein utilize parallel commercial solvers to solve for s_i in parallel (e.g., independently) at each height to be combined at a later height.

[0026] As described above AUD(T\X) corresponds to the audience of T when the audience of the union and/or margin X is removed from the tree structure, so the new graph only contains people who visited anything other than X. Because removing a union and/or margin X only affects the ancestors of X, AUD(T\X) can be determined based on the below-Equation 4. Utilizing Equation 4 reduces the number of computations for tree structure associations (e.g., the larger the number of nodes in the tree structure association, the larger the reduction of computations), because updating the ancestors to compute AUD(T\X) (e.g., using the below Equation 4) can be done in parallel, once for each node. Although new tree associations can be calculated for each node removal graph to determine the total audience of Equation 3, Equation 4 utilizes the property that the only s_i variables which are impacted by node deletion are the ancestors of that node.

[0027] $AUD(T\backslash X) = z_0 z_N s_N^{(new)}$ (Equation 4)

[0028] In Equation 4 above, $s_N^{(new)}$ is defined recursively by node height (e.g., margins being nodes at the lowest height, unions of margins being at the next height, etc.) when the X union and/or margin is removed. $s_N^{(new)}$ can be determined using the Equation 5, as shown below.

[0029] $$s_{k_j}^{(new)} = \begin{cases} \frac{s_{k_1}+1}{1+s_{k_0}z_{k_0}} - 1, & j = 1 \\ (\frac{s_{k_j}+1}{1+s_{k_{j-1}}z_{k-1}})(1 + s_{k_{j-1}}^{(new)} z_{k_{j-1}}) - 1, & j = 2, \ldots, J \end{cases}$$ (Equation 5)

[0030] In the preceding equation, k_j is the node representing the j^{th} ancestor of X (e.g., k_0 is X, k_1 is the parent of X, . . ., k_J is node N, the root node). As the calculations of s_{kj} depend on its children, if any, examples disclosed herein, when solving for s_{kj} can utilize parallel commercial solvers to solve for s_{kj} in parallel (e.g., independently) at each height to be combined at a later height.

[0031] Equation 1 (e.g., corresponding to Equations 2-5) results in a system of equations with variable Lagrange multipliers (λs). Examples disclosed herein determine the unique audience of one or more unions based on census data corresponding to the margins and panelist data corresponding to the margins and the unions by performing

three tasks corresponding to Equation 1. The first task includes solving a system of Equations corresponding to the below Equations 6 and 7 with respect to the panelist data.

[0032] AUG(T)-AUD(T\{i}) = A_i for all nodes of the tree structure association i = {1, ..., N} (Equation 6)

[0033] z_0 + AUG(T) = 100% (Equation 7)

[0034] In Equation 7 above, z_0 represents the total audience (e.g., panelist audience) count that was not exposed to the media. Equations 6 and 7 correspond to a system of equations with λs as solutions to the system. Estimates of AUD(T\{i}) (corresponding to the above Equations 4 and 5) can be determined using commercial solvers in parallel. The commercial solvers estimating values for the λs of AUD(T\{i}) and iteratively adjusting such that until all the constraints of Equations 6 and 7 are met. Because AUG(T) is known in panelist data, it follows that $\lambda_0=\ln(1-A_N)$, where A_N is AUD(T). Although it is assumed all quantities of audiences in the above Equations are expressed in percentages of the Universe Estimate (e.g., Equation 7 illustrates that for total population), Equations 1-7 may be used in conjunction with audience totals by slightly adjusting the Equations.

[0035] Once the first task is complete, examples disclosed herein perform a second task in which a first subset of the λs (e.g., λ_{M+1}-λ_N) corresponding to unions of the tree structure association are kept (e.g., the λs corresponding to unknowns in the census data) and a second subset of the λs (e.g., λ_0-λ_M) corresponding to margins of the tree structure association are discarded (e.g., the λs corresponding to knowns in the census data). However, different subsets of the λs may be kept and/or discarded based on the census data. For example, if the census data includes data related to a first margin, a second margin, a third margin, and a first union and is missing data related to a fourth margin and a second union, the subset of λs that are kept will correspond to the fourth margin and the second union and the subset of λs that are discarded will correspond to the first margin, the second margin, the third margin and the first union. In response to discarding the first set of λs, examples disclosed herein solve the system corresponding

to Equations 6 and 7 using the census data, the unknown Lagrange multipliers (e.g., λ_0-λ_M), and the known kept Lagrange multipliers of the first task (e.g., λ_{M+1}-λ_N), thereby resulting in a set of second task-based Lagrange multipliers $\hat{\lambda}$s based on both the panelist data and the census data. As used herein the hat notation (^) corresponds to the solutions to systems of equations based on the second task (e.g., including panelist and census data). Once the second task is complete, examples disclosed herein perform a third task to solve for the missing population unions using the below Equation 8.

[0036] $AUD(G)-(AUD(G\backslash\{j\}) = \hat{A}_j$ where $j= \{M+1, \ldots, N\}$ (Equation 8)

[0037] FIG. 1 is a block diagram of an environment in which example return path data 100 and example meter data 102 are collected to determine reach(es) based on marginal ratings data unions. FIG. 1 includes the example return path data 100, the example meter data 102, an example media provider 104, an example media presentation device 106, example media output devices 108, 110, an example local people meter (LPM) 112, and an example audience measurement entity (AME) 114. The example audience measurement entity 114 includes an example return path data (RPD) audience storage 116, an example panelist data storage 118, and an example union population reach determiner 120.

[0038] The example media provider 104 of FIG. 1 is a service provider (e.g., cable media service provider, a radio frequency (RF) media provider, a satellite media service provider, etc.) that presents media to an audience member via the example media presentation device 106. The media provided by the example media provider 104 is transmitted (e.g., via a wired or wireless network connection) to the media presentation device 106. The media presentation device 106 is connected, via a wired or wireless connection, to the example media output device 108 to output the media to an audience member. The media output device 108 is a device capable of outputting the received media. For example, the media output device 108 may be a television, a radio, speakers, a projector, a computer, a computing device, a tablet, a mobile device, and/or any other device capable of outputting media.

[0039] When the example media presentation device 106 of FIG. 1 is operating to access media, the media presentation device 106 receives media corresponding to a station, program, website, etc., based on the tuning of the example media presentation device 106. For example, the media presentation device 106 may be a set-top box. Additionally or alternatively, the example media presentation device 106 may be an over-the-top (OTT) device, a video game console, a digital video recorder (DVR), a digital versatile disc (DVD) player, a receiver, a router, a server, a computer, a mobile device, software executed by a website, computer, and/or application, and/or any device that receives media from a service provider. For example, the media presentation device 106 may be a website and/or application that provides media to users via the media output device 108. In some examples, the media presentation device 106 may implement a DVR and/or DVD player. In some examples, the example media presentation device 106 includes a unique serial number that, when associated with subscriber information, allows an audience measurement entity, a marketing entity, and/or any other entity to ascertain specific subscriber behavior information.

[0040] By way of example, the example media presentation device 106 may be tuned to channel 5. In such an example, the media presentation device 106 outputs media (from the example media provider 104) corresponding to the tuned channel 5. The media presentation device 106 may gather tuning data corresponding to which channels, stations, websites, etc., that the example media presentation device 106 was tuned. The example media presentation device 106 generates and transmits the example return path data 100 (e.g., census data corresponding to the total population of users) to the example media provider 104. The example return path data 100 includes the tuning data and/or data corresponding to the example media provider 104. Although the illustrated example of FIG. 1 includes the example media provider 104 receiving the example return path data 100 from one media presentation device (e.g., the example media presentation device 106), at one location, corresponding to one media provider (e.g., the example media provider 104), the example media provider 104 may receive return path data 100 from any number or type(s) of media presentation devices, at any number of

locations. The media provider 104 transmits the collected return path data 100 to the example audience measurement entity 114. Additionally or alternatively, the audience measurement entity 114 may be hosted by any other entity or may be co-hosted by another entity(ies). For example, the example return path data 100 may be collected from the example media presentation devices 106 by a media provider (e.g., a cable television provider, a satellite television provider, etc.) and the example meter data 102 may be collected from an LPM (e.g., such as the example LPM 112) by the example audience measurement entity 114 cooperating with the media provider to gain access to the tuning data. The example audience measurement entity 114 includes the example return path data audience storage 116 (e.g., a database) and the example panelist data storage 118 (e.g., a database).

[0041] The example media output device 110 of FIG. 1 is a device capable of outputting the received media. For example, the media output device 110 may be a television, a radio, speakers, a projector, a computer, a computing device, a tablet, a mobile device, and/or any other device capable of outputting media. In some examples, the media output device 110 receives media over-the-air. In this manner, the media output device 110 receives media via an antenna and does not correspond to a media provider (e.g., including the example media provider 104). In the illustrated example of FIG. 1, the media output device 110 corresponds to one or more monitored panelists. The example LPM 112 monitors the panelists exposure to media output by the example media output device 110. For example, the example LPM 112 is in communication with the example media output device 110 to collect and/or capture signals emitted externally by the media output device 110. The LPM 112 may be coupled with the media output device 110 via wired and/or wireless connection. The example LPM 112 may be implemented in connection with additional and/or alternative types of media presentation devices, such as, for example, a radio, a computer monitor, a video game console, and/or any other device capable of presenting media to a user. The LPM 112 may be a portable people meter, a cell phone, a computing device, a sensor, and/or any other device capable of metering (e.g., monitoring) user exposure to media. In some

examples, a media presentation location may include a group of LPMs 112. In such examples, the group of the LPMs 112 may be used to monitor media exposure for multiple users and/or media output devices 110. Additionally, the example panelist data storage 118 receives and stores the example meter data 102 from the example LPM 112.

[0042] In some examples, the example LPM 112 of FIG. 1 includes a set of buttons assigned to audience members to determine which of the audience members is watching the example media output device 110. The LPM 112 may periodically prompt the audience members via a set of LEDs, a display screen, and/or an audible tone, to indicate that the audience member is present at a first media presentation location by pressing an assigned button. In some examples, to decrease the number of prompts and, thus, the number of intrusions imposed upon the media consumption experience of the audience members, the LPM 112 prompts only when unidentified audience members are located at the first media presentation location and/or only after the LPM 112 detects a channel change and/or a change in state of the media output device 110. In other examples, the LPM 112 may include at least one sensor (e.g., a camera, 3-dimensional sensor, etc.) and/or be communicatively coupled to at least one sensor that detects a presence of the user in a first example media presentation location. The example LPM 112 transmits the example meter data 102 to a media researcher and/or a marketing entity. The example meter data 102 includes the media presentation data (e.g., data related to media presented while the media output device 110 is on and a user is present). The example meter data 102 may further include a household identification, a tuner key, a presentation start time, a presentation end time, a channel key, etc. Although the illustrated example illustrates the example audience measurement entity 114 collecting the example meter data 102 from one LPM 112 at one location, the example audience measurement entity 114 may collect meter data from any number or type of meters at any number of locations.

[0043] The example return path data 100 of FIG. 1 from the example media presentation device 106 and/or the example meter data 102 from the example LPM 112 is transmitted to the example audience measurement entity 114 via a network. The

network may be implemented using any type of public or private network, such as, but not limited to, the Internet, a telephone network, a local area network (LAN), a cable network, and/or a wireless network. To enable communication via the network, the example media presentation device 106 includes a communication interface that enables a connection to an Ethernet medium, a digital subscriber line (DSL), a telephone line, a coaxial cable, or any wireless connection, etc.

[0044] The example return path data audience storage 116 of the example AME 114 of FIG. 1 collects the example return path data 100 corresponding to the example media presentation device(s) 106. As described above, the example return path data 100 includes tuning data of the example media presentation device 106. However, in some examples, the example return path data 100 may not include specific data identifying any information relating to the audience of the example media output device 108. In such examples, another device and/or processor models such audience information prior to storing in the example return path data audience storage 116. For example, the device and/or processor may assign and/or model virtual users to augment the example return path data 100, thereby generating audience assigned return path data.

[0045] The example union population reach determiner 120 of FIG. 1 receives the recorded audience marginal ratings for different time-periods (e.g., quarter hours, half hours, hours, days, etc.) of the media from the example panelist data storage 118, the population (e.g., based on census data) marginal ratings for the different marginals (e.g., time periods) of the media from the RPD audience data storage 116, the total reach of the recorded audience from the different marginal ratings from the panelist data storage 118, and a total population size (e.g., the universe estimate) from one or more devices from either the RPD audience data storage 116, the panelist data storage 118, and/or any other storage. Additionally, the example union population reach determiner 120 may receive union data related to unions of the different marginals and/or may be structured to develop preset unions based on margins of a particular media. For example, the union population reach determiner 120 may receive unions based on instructions from a user and/or a manufacturer. The received unions may be representative of half hour, hour,

daypart, and/or any combination of time interval corresponding to some or all of the quarter hours for a particular day. In some examples, a union may include multiple unions. For example, union ABCD (e.g., 15 minute margins of an hour show) may be a union of union AB (e.g., a first union of the first two 15 minute increments representative of the unique panel audience of the first 30 minutes of the show) and union CD (e.g., a first union of the first two 15 minute increments representative of the unique panel audience of the second 30 minutes of the show), where union AB is a union of marginal A and marginal B and union CD is a union of marginal C and marginal D. Each union may have corresponding descendants and/or ancestors. Using the above example, the parent of union AB is union ABCD and the children of union AB are A and B. Each union corresponds to a union reach (e.g., a deduplicated number of people exposed to media across all marginals in the union). The example union population reach determiner 120 generates a tree structure association by tagging each node (e.g., each margin or union) with the corresponding descendants and/or ancestors. The example union population reach determiner 120 determines the unique panelist audience count percentage by processing the received panelist data. For example, if the union population reach determiner 120 is to determine a reach across two margins, A and B, the union population reach determiner 120 will process the panelist counts corresponding to the reach of A and the panelist counts corresponding to the reach of B to determine the duplication and determine the reach across AB to be the sum of A and B minus the duplication.

[0046] Once the example union population reach determiner 120 of FIG. 1 determines the panelist reach values of the unions, the union population reach determiner uses the above Equations 1-7 to perform three tasks to determine the reach(es) of one or more of the unions for the population. The union population reach determiner 120 executes the first task by utilizing commercial solvers to determine λs from a system of equations corresponding to Equations 6 and 7 based on the panelist data. The union population reach determiner 120 executes the second task by utilizing commercial solvers to discard a first subset of the λs corresponding to known census

data from the first task (e.g., returning the corresponding λs to variables) and keep a second subset of the λs corresponding to unknown census data from the first task. For example, if the census data is known for the margins and the census data is unknown for the unions, the union population reach determiner 120 executes the second task by utilizing commercial solvers to discard a first subset of the λs corresponding to the margins of the first task and keep a second subset of the λs corresponding to the union of the first task. The union population reach determiner 120 determines the second task-based Lagrange multipliers λ̂s (e.g., corresponding to the now variable λs corresponding to the margins) from a system of equations corresponding to Equations 6 and 7 based on the second subset of the λs from the first task and the census data corresponding to the margins. The union population reach determiner 120 executes the third task by solving for the missing population unions using $AUD(G)-(AUD(G\backslash\{j\}) = \hat{A}_j$ where j= {M+1, . . ., N}, where M+1, . . ., N represent the nodes corresponding to the discarded first subset of λs from the second task. The example union population reach determiner 100 is further described below in conjunction with FIG. 3.

[0047] FIGS. 2A-C illustrates an example transformation of example marginal and union data and a corresponding representation of a tree structure association that may be generated by the example union population reach determiner 120 of FIG. 1. FIG. 2A includes example marginal and union data 200. FIG 2B includes an example tree structure 202. FIG. 2C includes an example tree structure conversion 204 corresponding to a removal of a node, and an example node removal tree structure 206.

[0048] The example marginal and union data 200 of FIG. 2A corresponds to the percentage of the panel and/or the population that was exposed to media at different margins and/or unions based on the return path data 100 stored in the example RPD audience data storage 116 and the meter data 102 stored in the example panelist data storage 118. For example, the union population reach determiner 120 may be instructed (e.g., based on user, manufacturer, and/or preset instructions) to generate reaches (e.g., a unique deduplicated audience for the population) across union AB, union CD, and union ABCD. In such an example, the union population reach determiner 120 accesses the

panelist data corresponding to the margins that make up the unions to determine the percentage of panelists that were exposed to the media at margin A, B, C, D and the percentage of panelists that were exposed to the media at unions AB, BC, and ABCD. Accordingly, the union population reach determiner 120 processes the panelist data storage 118 to remove duplicate panelist data for the unions (e.g., panelist that were included more than one margin corresponding to a union).

[0049] For example, as shown in the marginal and union data 200 of FIG. 2A, the union population reach determiner 120 determines that 10% (e.g., 0.1) of the panelists were exposed to the media during margin A (e.g., a first 15-minute duration of an hour program), 20% of the panelists were exposed to the media during margin B (e.g., a second 15-minute duration of the hour program), 30% of the panelists were exposed to the media during margin C (e.g., a third 15-minute duration of the hour program), 40% of the panelists were exposed to the media during margin D (e.g., a fourth 15-minute duration of the hour program), 25% of the panelists were exposed to the media during union AB (e.g., a first 30-minute duration of the hour program including the first and second 15-minute durations), 55% of the panelists were exposed to the media during union CD (e.g., a second 30-minute duration of the hour program including the third and fourth 15-minute durations), and 70% of the panelists were exposed to the media during union ABCD (e.g., the entire duration of the hour program including the first and second 30-minute durations). Additionally, the union population reach determiner 120 obtains census data corresponding to the percentage of the population that corresponds to the exposure of the media at the margins A, B, C, and D based on the return path data 100. The census data may be stored in the example RPD audience data storage 116 as a percentage of the population exposed to the media at the margin or may be a number of persons exposed to the media at the margin. When the census data corresponds to a number of persons, the union population reach determiner 120 may convert the number into a percentage by dividing the number of persons by the total population or total population estimate. In the example of FIG. 2A, the union population reach determiner 120 determines that 20% of the population of users was exposed to the first margin A,

40% of the population of users was exposed to the second margin B, 60% of the population of users was exposed to the third margin C, and 80% of the population of users was exposed to the fourth margin D.

[0050] The example tree structure 202 of FIG. 2B corresponds to the tree structure of the margins and unions of the example marginal and union data 200. The tree structure 202 corresponds to a tree linkage of margins and unions based on the unions identified in the example margin and union data 200. Alternatively, other tree structures can be generated based on different combination of margins and/or unions (e.g., so long as each node has no more than one parent). The example union population reach determiner 120 generates a tree structure association or tree associations corresponding to the example tree structure 202 by tagging the margins and/or unions with corresponding node numbers, ancestors, and/or descendants. For example, the union population reach determiner 120 may tag the AB union with a node number (e.g., 5) and may tag the AB node as having a parent union of ABCD (e.g., node 7) and children margins A and B (e.g., nodes 1 and 2). In this manner, commercial solvers of the union population reach determiner 120 can utilize the values of the example margin and union data 200 for the corresponding variables of the above Equations 1-7.

[0051] The example tree structure conversion 204 of FIG. 2C illustrates a removal of a node (e.g., node 6 corresponding to union CD) from the example tree structure 202, thereby resulting in the example node removal tree structure 206. The example tree structure 202 corresponds to the total audience (e.g., AUD(T) of Equations 1 and 6) while the example node removal tree structure 206 corresponds to the total audience if node 6 (e.g., union CD) was removed (e.g., AUD(T\6) of Equations 1 and 6). The example union population reach determiner 120 can compute AUD(T/6) using a first technique by utilizing the example node removal tree structure 206 itself and utilize the above Equation 3 or using a second technique by computing s values of only the ancestors that are impacted (e.g., using Equation 5), thereby conserving memory and time by eliminating computations corresponding to the first technique. The example union population reach determiner 120 can determine the population audience of union

CD by leveraging the fact that the audience of CD is equivalent to the audience of ABCD minus the audience of ABCD if the CD union is removed, as shown in the example node removal tree structure 206.

[0052] FIG. 3 is a block diagram of an example implementation of the example union population reach determiner 120 of FIG. 1 to determine population reach for given media based on unions of marginal ratings data. The example union population reach determiner 120 includes an example interface(s) 300, an example data converter 302, an example association controller 304, an example local memory 306, and an example commercial solver(s) 308. Although the example union population reach determiner 120 is described in conjunction with return path data, the example union population reach determiner 120 may determine reach based on information provided by any device that is capable of inputting audience marginal ratings, population marginal ratings, audience reach from the unions, and universe estimates. For example, the example union population reach determiner 120 may determine one or more reaches of from different programs, store visits for different stores, website visits, etc.

[0053] The example interface(s) 300 of FIG. 3 receives recorded audience (e.g., panelist) marginal ratings corresponding to media for two or more marginals (e.g., quarter hours), population audience marginal ratings corresponding to the media for the two or more marginals, total recorded audience reach for the two or more marginal ratings, and/or a universe estimate of users. In some examples, the interface(s) 300 receives the recorded audience (e.g., panelist) marginal data and the data converter 302 uses the recorded audience marginal data to determine the total recorded audience reach by deduplicating the panelists in both margins and/or unions of a union. In some examples, the interface(s) 300 receives recorded audience totals and population audience totals for predetermined margins and the data converter 302 converts the totals to percentages based on the respective total audience populations. The marginals may be broken up into quarter hours. For example, the recorded and/or population audience marginal ratings may correspond to four quarter hour marginals of an hour media program. However, the marginals may be broken up into any increments of time.

Additionally, the example interface(s) 300 may output an estimated population reach (e.g., a deduplicated population audience total) for one or more unions to another device/system and/or as a report to a user.

[0054] The example data converter 302 of FIG. 3 processes the return path data from the RPD audience data storage 116 and/or the meter data from panelist data storage 118. The example data converter 302 selects one or more unions to analyze (e.g., to determine population reach(es)). The data converter 302 may be structured to select preset union(s) and/or may receive instructions from a user and/or a manufacturer corresponding to one or more union(s) to analyze. For example, in the example described in conjunction with FIG. 2A, the data converter 302 selects unions AB, CD, and ABCD from the margins A, B, C, and D. Alternatively, different unions based on different margins may be used so long as each union and/or margin only has one parent. Using the above example, the data converter 302 obtains return path data from the RPD audience data storage 116 via the interface(s) 300 corresponding to the margins A, B, C, and D. If the return path data stored in the RPD audience data storage 116 corresponds to audience (e.g., exposed to the media) population totals of the margins, the data converter 302 converts the audience population totals of the margins by dividing the audience population totals with the universe estimate (e.g., the total population) to generate an audience population percentage for the margins. Additionally, the data converter 302 obtains meter data from the panelist data storage 118 corresponding to the margins (e.g., A, B, C, D). If the panelist data storage 118 stores deduplicated panelist data for the determined unions, the data converter 302 obtains the meter data from the panelist data storage 118 corresponding to the determined unions (e.g., AB, CD, ABCD). If the panelist data storage 118 does not store deduplicated panelist data for the determined unions, the data converter 302 obtains the meter data from the panelist data storage 118 corresponding to the margins (e.g., A, B, C, and D), and processes the meter data to determine duplicate panel members included in both margins of a union and removes the duplicates from the sum of the totals of the margins, thereby generating a deduplicated panelist total for the union. Additionally, the data converter 302 converts

the panelist audience totals of the margins and/or unions by dividing the panelist count audience totals with the panelist universe estimate (e.g., the total number of panelists) to generate a panelist audience percentage for the margins and/or unions.

[0055] The example association controller 304 of FIG. 3 generates the tree structure association based on the margins and selected unions. For example, using the unions and margins of the example of FIG. 2A, the association controller 304 tags each margin and union with (i) a number or other identifier and (ii) with corresponding ancestors and/or descendants. For example, the association controller 304 tags the AB union (e.g., node of the tree structure 202 of FIG. 2B) with a node number/identifier (e.g., 5) and tags the AB node/union as having a parent union of ABCD (e.g., node 7) and children margins A and B (e.g., nodes 1 and 2). The association controller 304 stores the tags in conjunction with the panelist identifiers and/or population margin and/or union percentages in the example local memory 306. In this manner, the commercial solver(s) 308 can solve a system of equations using the tagged panelist identifier and/or population margin and/or union percentages stored in the example local memory 306.

[0056] The example commercial solver(s) 308 of FIG. 3 are optimization software packet(s) that solve(s) one or more system of equations using the tagged panelist identifier and/or population margin and/or union percentages stored in the example local memory 306 to estimate the population reach of one or more union(s) of the margins. For example, the commercial solver(s) 308 may be a CPLEX optimizer, a GNU linear programming kit (GLPK), a Gurobi Optimizer, a solving constraint integer program, and/or any type of mixed integer programming optimizer. As described above, the commercial solver(s) 308 execute(s) three tasks (e.g., corresponding to solving for parameters in a system of equations) to estimate the population reach(es). The first task results in parameters of the panel, the second task results in parameters of the population, and the third task determines the one or more population reaches based on the parameters of the first task and the second task. For example, the commercial solver(s) 308 solve(s) the system of equations using the above Equation 6 and Equation 7 with an iterative process utilizing the tagged panelist union and margin data to solve

for Lagrange multipliers (e.g., λs) that satisfy the system of Equations 6 and 7. Based on Equation 6, the example commercial solver(s) 308 need to determine AUD(T\{i}) for all nodes i. Because removing node i only affects ancestors of i, the example commercial solver(s) 308 performs parallel computations with a processor to solve for the λs corresponding to AUD(T\{i}) in parallel at each height starting at the lowest height (e.g., the margin height in parallel first, the parent height in parallel second, the grandparent height in parallel third, etc.). For example, the commercial solver(s) 308 can solve for the λs in parallel by performing computations with a processor (e.g., concurrently processing threads in a multi-threading processor, utilizing multiple cores of a multiple ore processor in parallel, utilizing a multiple processor system in parallel, etc.). Completion of the first task results in a set of λs, each corresponding to nodes of the tree structure. Once the first task is complete, the example commercial solver(s) 308 perform(s) a second task in which a first subset of the λs (e.g., λ_0-λ_M) corresponding to known census data (e.g., the margins in the example of FIG. 2A) of the tree structure association are discarded and a second subset of the λs (e.g., λ_{M+1}-λ_N) corresponding to unknown census data (e.g., the unions in the example of FIG. 2A) of the tree structure association are kept. In response to discarding the first subset of λs, the example commercial solver(s) 308 solve(s) the system corresponding to Equations 6 and 7 using the census data (e.g., in parallel at the height level), the unknown variable (e.g., λ_0-λ_M), and the known values of the first task (e.g., λ_{M+1}-λ_N), thereby resulting in a set of second task-based Lagrange multipliers λ̂s based on both the panelist data and the census data. Using the example of FIG. 2A, the commercial solver(s) 308 discard the subset of λs that correspond to nodes 1, 2, 3, and 4 (e.g., margins A, B, C, and D) and keeps the subset of λs that correspond to nodes 5, 6, and 7 (e.g., unions AB, CD, and ABCD). After the discarding of the corresponding λs, the example commercial solver(s) 308 solve(s) the system of Equations based on Equations 6 and 7 using the maintained λs and the census data, thereby resulting in a set of second task-based Lagrange multipliers λ̂s (e.g., including the maintained λs and additional λs solved during the second task) based on both the panelist data and the census data. Once the second task is complete, the

example commercial solver(s) 308 performs a third task using the set of second task-based Lagrange multipliers $\hat{\lambda}$s in conjunction with the above Equation 8 to determine the population reach estimate(s) for one or more union(s).

[0057] While an example manner of implementing the example population reach determiner 120 of FIG. 1 is illustrated in FIG. 3, one or more of the elements, processes and/or devices illustrated in FIG. 3 may be combined, divided, re-arranged, omitted, eliminated and/or implemented in any other way. Further, the example interface(s) 300, the example data converter 302, the example association 304, the example local memory 306, the example commercial solver(s) 308, and/or, more generally, the example population reach determiner 120 of FIG. 3 may be implemented by hardware, software, firmware and/or any combination of hardware, software and/or firmware. Thus, for example, any of the example interface(s) 300, the example data converter 302, the example association 304, the example local memory 306, the example commercial solver(s) 308, and/or, more generally, the example population reach determiner 120 of FIG. 3 could be implemented by one or more analog or digital circuit(s), logic circuits, programmable processor(s), programmable controller(s), graphics processing unit(s) (GPU(s)), digital signal processor(s) (DSP(s)), application specific integrated circuit(s) (ASIC(s)), programmable logic device(s) (PLD(s)) and/or field programmable logic device(s) (FPLD(s)). When reading any of the apparatus or system claims of this patent to cover a purely software and/or firmware implementation, at least one of example interface(s) 300, the example data converter 302, the example association 304, the example local memory 306, the example commercial solver(s) 308, and/or, more generally, the example population reach determiner 120 of FIG. 3 is/are hereby expressly defined to include a non-transitory computer readable storage device or storage disk such as a memory, a digital versatile disk (DVD), a compact disk (CD), a Blu-ray disk, etc. including the software and/or firmware. Further still, the example population reach determiner 120 of FIG. 3 may include one or more elements, processes and/or devices in addition to, or instead of, those illustrated in FIG. 3, and/or may include more than one of any or all of the illustrated elements, processes and devices.

As used herein, the phrase "in communication," including variations thereof, encompasses direct communication and/or indirect communication through one or more intermediary components, and does not require direct physical (e.g., wired) communication and/or constant communication, but rather additionally includes selective communication at periodic intervals, scheduled intervals, aperiodic intervals, and/or one-time events.

[0058] Flowcharts representative of example hardware logic, machine readable instructions, hardware implemented state machines, and/or any combination thereof for implementing the example population reach determiner 120 of FIG. 1 are shown in FIGS. 4-7. The machine readable instructions may be one or more executable programs or portion(s) of an executable program for execution by a computer processor such as the processor 812 shown in the example processor platform 800 discussed below in connection with FIG. 8. The program(s) may be embodied in software stored on a non-transitory computer readable storage medium such as a CD-ROM, a floppy disk, a hard drive, a DVD, a Blu-ray disk, or a memory associated with the processor 812, but the entireties of the program(s) and/or parts thereof could alternatively be executed by a device other than the processor 812 and/or embodied in firmware or dedicated hardware. Further, although the example program(s) is/are described with reference to the flowcharts illustrated in FIGS. 4-7, many other methods of implementing the example population reach determiner 120 may alternatively be used. For example, the order of execution of the blocks may be changed, and/or some of the blocks described may be changed, eliminated, or combined. Additionally or alternatively, any or all of the blocks may be implemented by one or more hardware circuits (e.g., discrete and/or integrated analog and/or digital circuitry, an FPGA, an ASIC, a comparator, an operational-amplifier (op-amp), a logic circuit, etc.) structured to perform the corresponding operation without executing software or firmware.

[0059] The machine readable instructions described herein may be stored in one or more of a compressed format, an encrypted format, a fragmented format, a packaged format, etc. Machine readable instructions as described herein may be stored as data

(e.g., portions of instructions, code, representations of code, etc.) that may be utilized to create, manufacture, and/or produce machine executable instructions. For example, the machine readable instructions may be fragmented and stored on one or more storage devices and/or computing devices (e.g., servers). The machine readable instructions may require one or more of installation, modification, adaptation, updating, combining, supplementing, configuring, decryption, decompression, unpacking, distribution, reassignment, etc. in order to make them directly readable and/or executable by a computing device and/or other machine. For example, the machine readable instructions may be stored in multiple parts, which are individually compressed, encrypted, and stored on separate computing devices, wherein the parts when decrypted, decompressed, and combined form a set of executable instructions that implement a program such as that described herein. In another example, the machine readable instructions may be stored in a state in which they may be read by a computer, but require addition of a library (e.g., a dynamic link library (DLL)), a software development kit (SDK), an application programming interface (API), etc. in order to execute the instructions on a particular computing device or other device. In another example, the machine readable instructions may need to be configured (e.g., settings stored, data input, network addresses recorded, etc.) before the machine readable instructions and/or the corresponding program(s) can be executed in whole or in part. Thus, the disclosed machine readable instructions and/or corresponding program(s) are intended to encompass such machine readable instructions and/or program(s) regardless of the particular format or state of the machine readable instructions and/or program(s) when stored or otherwise at rest or in transit.

[0060] As mentioned above, the example processes of FIGS. 4-7 may be implemented using executable instructions (e.g., computer and/or machine readable instructions) stored on a non-transitory computer and/or machine readable medium such as a hard disk drive, a flash memory, a read-only memory, a compact disk, a digital versatile disk, a cache, a random-access memory and/or any other storage device or storage disk in which information is stored for any duration (e.g., for extended time

periods, permanently, for brief instances, for temporarily buffering, and/or for caching of the information). As used herein, the term non-transitory computer readable medium is expressly defined to include any type of computer readable storage device and/or storage disk and to exclude propagating signals and to exclude transmission media.

[0061] "Including" and "comprising" (and all forms and tenses thereof) are used herein to be open ended terms. Thus, whenever a claim employs any form of "include" or "comprise" (e.g., comprises, includes, comprising, including, having, etc.) as a preamble or within a claim recitation of any kind, it is to be understood that additional elements, terms, etc. may be present without falling outside the scope of the corresponding claim or recitation. As used herein, when the phrase "at least" is used as the transition term in, for example, a preamble of a claim, it is open-ended in the same manner as the term "comprising" and "including" are open ended. The term "and/or" when used, for example, in a form such as A, B, and/or C refers to any combination or subset of A, B, C such as (1) A alone, (2) B alone, (3) C alone, (4) A with B, (5) A with C, (6) B with C, and (7) A with B and with C. As used herein in the context of describing structures, components, items, objects and/or things, the phrase "at least one of A and B" is intended to refer to implementations including any of (1) at least one A, (2) at least one B, and (3) at least one A and at least one B. Similarly, as used herein in the context of describing structures, components, items, objects and/or things, the phrase "at least one of A or B" is intended to refer to implementations including any of (1) at least one A, (2) at least one B, and (3) at least one A and at least one B. As used herein in the context of describing the performance or execution of processes, instructions, actions, activities and/or steps, the phrase "at least one of A and B" is intended to refer to implementations including any of (1) at least one A, (2) at least one B, and (3) at least one A and at least one B. Similarly, as used herein in the context of describing the performance or execution of processes, instructions, actions, activities and/or steps, the phrase "at least one of A or B" is intended to refer to implementations including any of (1) at least one A, (2) at least one B, and (3) at least one A and at least one B.

[0062] FIG. 4 is an example flowchart 400 representative of example machine readable instructions that may be executed by the example union population reach determiner 120 of FIGS. 1 and/or 3 to generate respective population reach estimates for various unions. The example flowchart 400 is described in conjunction with the example marginal and union data 200 of FIG. 2A. However, the example flowchart 400 may be implemented in conjunction with any panelist data, census data, margins, and/or unions.

[0063] At block 401, the example interface(s) 300 (FIG. 3) obtains panelist data and census data of the margins and/or unions of media. In some examples, the association controller 304 (FIG. 3) instructs the interface(s) 300 to access the databases 116, 118 of FIG. 1 to obtain particular marginal and/or union data based on predefined margins and/or unions (e.g., the predefined margins and/or unions based on user and/or a manufacturer preferences). Using the example of FIG. 2A, the interface(s) 300 obtains census and/or panelist data from the example RPD audience data storage 116 corresponding to particular media (e.g., a television show) at margins A, B, C, and D and obtains panelist data from the example panelist data storage 118 corresponding to the particular media at margins A, B, C, and D and/or unions AB, BD, and ABCD (if available).

[0064] At block 402, the example data converter 302 (FIG. 3) processes the panelist data and the census data of the margins and/or unions of the media to determine the panelist margin and union percentages and the population margin percentages, as represented in the example marginal and union data 200 of FIG. 2A. As described above in conjunction with FIG. 3, the data converter 302 may convert marginal and/or union totals to marginal percentages (e.g., by dividing the totals by respective universe estimates) and/or may determine union percentages by deduplicating duplicate panelist counts in a union. At block 403, the example local memory 306 (FIG. 3) stores the marginal percentages and/or union percentages based on the panelist data and census data. At block 404, the example association controller 304 generates a tree structure association of the unions/margins. As described above in conjunction with FIG. 3, the

example association controller 304 (FIG. 3) generates a tree structure association by tagging the margins and unions with numbers and/or identifiers and tagging each node with corresponding ancestor and/or descendant information. The tree structure association information is stored in the example local memory 306.

[0065] At block 406, the example commercial solver(s) 308 (FIG. 3) determines(s) Lagrange multipliers based on panel parameters. For example, the commercial solver(s) 308 solve(s) for the Lagrange multipliers (λ_0=-1.20397 , λ_1=-1.09861, λ_2=0, λ_3-0.510826 , λ_4=0, λ_5=-2.0149, λ_6=-1.19392, λ_7=0.81093) which corresponds to the solution for all panelist constraints (e.g., AUD(A) = .1, AUD(B) = 0.2, AUD(C) = 0.3, AUD(D) = 0.4, AUD(AB) = 0.25, AUD(CD) = 0.55, AUD(ABCD) = 0.7) using Equations 6 and 7 based on the panelist data and tree structure association data stored in the local memory 306 (FIG. 3). An example process that may be used to implement block 406 is described below in conjunction with FIG. 5. For example, the commercial solver(s) 308 determine(s) that to satisfy the system of equations represented by Equations 6 and 7, λ_0 = -1.20397, λ_1 = -1.09861, λ_2 = 0, λ_3 = -0.510826, λ_4 = 0, λ_5 = -2.0149, λ_6 = -1.19392, and λ_7 = 0.81093, where λ_0 corresponds to the normalized constraint, λ_1 corresponds to the margin A (e.g., corresponding to node 1), λ_2 corresponds to the margin B (e.g., corresponding to node 2), . . . , and λ_7 corresponds to the union ABCD (e.g., corresponding to node 7). Additionally, because $z_i = e^{\lambda_i}$, the commercial solver(s) 308 determine(s) that z_0 = 0.3, z_1 = 0.333, z_2 = 1, z_3 = 0.6, z_4 = 1, z_5 = 0.133, z_6 = 0.303, z_7 = 2.25.

[0066] At block 408, the example commercial solver(s) 308 keep(s) values of a first subset of the Lagrange multipliers corresponding to the unknown census data (e.g., λ_5 = -2.0149, λ_6 = -1.19392, λ_7 = 0.81093, corresponding to the union nodes 5, 6, and 7 of the tree structure 202 of FIG. 2B). At block 410, the example commercial solver(s) 308 discard(s) a second subset of the Lagrange multipliers corresponding to known census data (e.g., the margins in the example of FIG. 2A). For example, because the census data is known for margin A, B, C, D and the audience total (e.g., corresponding to nodes

0, 1, 2, 3, and 4), the example commercial solver(s) 308 discard(s) the corresponding Lagrange multipliers, thereby designating the Lagrange multiplier variables λ_0, λ_1, λ_2, λ_3, and λ_4 as empty or ready to be determined at a subsequent time.

[0067] At block 412, the example commercial solver(s) 308 determine(s) the variable Lagrange multipliers (e.g., λ_0, λ_1, λ_2, λ_3, and λ_4) based on census parameters. For example, the commercial solver(s) solve(s) for the Lagrange multipliers (e.g., λ_0, λ_1, λ_2, λ_3, and λ_4) using the system of equations corresponding to Equations 6 and 7 based on the population parameters (e.g., AUD(A) = 0.2, AUD(B) = 0.4, AUD(C) = 0.6, AUD(D) = 0.8) and the first subset of the Lagrange multipliers (e.g., λ_5 = -2.0149, λ_6 = -1.19392, λ_7 = 0.81093), thereby resulting in second task-based Lagrange multipliers $\hat{\lambda}$s (e.g., the values of λ_0, λ_1, λ_2, λ_3, and λ_4 from block 412 and the values of λ_5, λ_6, and λ_7 from block 406) as further described below in conjunction with FIG. 6. For example, using the example data of FIG. 2A, the commercial solver(s) 308 determine that $\hat{\lambda}_0$ = -3.17489, $\hat{\lambda}_1$ = -0.498689, $\hat{\lambda}_2$ = 1.12924, $\hat{\lambda}_3$ = -0.574063, $\hat{\lambda}_4$ = 1.7578, $\hat{\lambda}_5$ = -2.0149, $\hat{\lambda}_6$ = -1.19392, and $\hat{\lambda}_7$ = 0.81093. Additionally, because $z_i = e^{\lambda_i}$, the commercial solver(s) 308 determine(s) that $\hat{z}_0$ = 0.0417986, $\hat{z}_1$ = 0.607326, $\hat{z}_2$ = 3.09329, $\hat{z}_3$ = 1.77547, $\hat{z}_4$ = 5.79965, $\hat{z}_5$ = 0.1333, $\hat{z}_6$ = 0.303, $\hat{z}_7$ = 2.25. At block 414, the example commercial solver(s) 308 estimate the population reach(es) for one or more of the unions (e.g., AB, CD, and/or ABCD) based on the Lagrange multipliers (e.g., and/or the example $\hat{z}$ values) determined at blocks 406 and 412 using the above Equation 8, as further described below in conjunction with FIG. 7. As a result of block 414, the example commercial solver(s) 308 determines that the population reach percentage for the AB union is 0.44886 of the universe estimate of audience members, the population reach of the CD union is 0.88824 of the universe estimate of the audience members, and the population reach of the ABCD union is 0.958201 of the universe estimate of the audience members.

[0068] FIG. 5 is an example flowchart 406 representative of example machine readable instructions that may be executed by the example union population reach determiner 120 of FIGS. 1 and/or 3 to solve for Lagrange multipliers based on the panel

parameters (e.g., AUD(A) = 0.1, AUD(B) = 0.2, AUD(C) = 0.3, AUD(D) = 0.4, AUD(AB) = 0.25, AUD(CD) = 0.55, AUD(ABCD) = 0.7). The instructions of FIG. 5 may be used to implement block 406 of FIG. 4. The example flowchart 406 is described in conjunction with the example marginal and union data 200 of FIG. 2A. However, the example flowchart 406 may be implemented in conjunction with any panelist data, census data, margins, and/or unions.

[0069] At block 502, the example commercial solver(s) 308 (FIG. 3) determine(s) the panelist audience percentage across all margins (e.g., AUD(T) of Equation 6). Using the example of FIG. 2A, the commercial solver(s) 308 determine(s) that the panelist audience across all margins corresponds to 0.7 or 70% (e.g., the percentage corresponding to the union ABCD). At block 504, the example commercial solver(s) 308 determine(s) the total percentage of panelists not exposed to media (e.g., z_0 of Equation 7). For example, the commercial solver(s) 308 determine(s) that the total percentage of panelists not exposed to media is 0.3 (e.g., z_0 +0.7 = 1.0, from Equation 7). Accordingly, the commercial solver(s) 308 determine(s) that λ_0 = -1.20397, because $z_i = e^{\lambda_i}$.

[0070] At block 506, the example commercial solver(s) 308 set(s) up a system of equations based on Equation 6 corresponding to removal off each node of the tree structure association using the panelist data. For example, the commercial solver(s) 308 set(s) up a system of equations including 0.7 – AUD(T\1) = 0.1 for node 1 (e.g., margin A), 0.7 – AUD(T\2) =0.2 for node 2 (e.g., margin B), . . . , and 0.7 – AUD(T\6) = 0.55 for node 6 (e.g., union CD).

[0071] At block 510, the example commercial solver(s) 308 select(s) all equations corresponding to node(s) at the lowest available height (e.g., margins being the lowest height and the union corresponding to the total representing the highest height). As described above, because removing a node from the tree structure association only affects the ancestors of the node, the commercial solver(s) 308 performs parallel computations with a processor to can calculate the Lagrange multipliers at the same

height in parallel without affecting the other nodes. For example, initially the commercial solver(s) select(s) the equations corresponding to the lowest level nodes (e.g., 1, 2, 3, and 4) corresponding to the margins A, B, C, and D (e.g., 0.7 – AUD(T\1) = 0.1, 0.7 – AUD(T\2) =0.2, 0.7 – AUD(T\3) =0.3, and 0.7 – AUD(T\4) =0.3).

[0072] At block 510, the example commercial solver(s) 308 select(s) value(s) for the Lagrange multiplier(s) that satisfy(ies) the equations. As described above, each of the selected equations corresponds to Equation 4, which include variable(s) that can be identified using Equation 5. Because both Equation 4 and Equation 5 include z values and $z_i = e^{\lambda i}$, the commercial solver(s) 308 can select a value for the Lagrange multipliers (λi) that satisfies the selected equation(s). The example commercial solver(s) 308 determine if the selected value(s) satisfy(ies) the constraint(s) of the selected equation(s) (e.g., within a threshold amount of error) (block 512).

[0073] If the example commercial solver(s) 308 determine(s) that the selected value(s) do(es) not satisfy(ies) the constraint(s) of the selected equation(s) (block 512: NO), the commercial solver(s) 308 adjust(s) the selected value(s) of the Lagrange multipliers (block 514), and the control returns to block 512 until the selected value(s) do(es) satisfy the constraints (e.g., within the threshold amount of error). In this manner, the example commercial solver(s) 308 perform(s) parallel computations with a processor to determine the Lagrange multipliers in an iterative fashion in parallel. If the example commercial solver(s) 308 determine(s) that the selected value(s) do(es) satisfy the constraint(s) of the selected equation(s) (block 512: YES), the example commercial solver(s) 308 determine(s) if there are additional node(s) of the tree structure association at additional height(s) (block 516). For example, once the example commercial solver(s) 308 determine(s) the Lagrange multipliers for the margins A, B, C, D, the commercial solver(s) 308 determine(s) that there are additional nodes (e.g., AB and CD) at the next lowest height. If the commercial solver(s) 308 determine(s) that there are additional node(s) at additional height(s) (block 516: YES), the control returns to block 508 to determine the Lagrange multipliers corresponding to the additional node(s). If

the commercial solver(s) 308 determine(s) that there are not additional node(s) at additional height(s) (block 516: NO), the example process of FIG. 5 ends, and control returns to a calling function or process such as the example process of FIG. 4.

[0074] FIG. 6 is an example flowchart 412 representative of example machine readable instructions that may be executed by the example union population reach determiner 120 of FIGS. 1 and/or 3 to solve for the variable Lagrange multipliers based on population parameters (e.g., AUD(A) = 0.2, AUD(B) = 0.4, AUD(C) = 0.6, AUD(D) = 0.5) and the first subset of Lagrange multipliers (e.g., λ_5 = -2.0149, λ_6 = -1.19392, λ_7 = 0.81093) of block 408 of FIG. 4. The example instructions of FIG. 6 may be used to implement block 412 of FIG. 4. The example flowchart 412 is described in conjunction with the example marginal and union data 200 of FIG. 2A. However, the example flowchart 412 may be described in conjunction with any panelist data, census data, margins, and/or unions.

[0075] At block 602, the example commercial solver(s) 308 set(s) up a system of equations based on Equation 6 corresponding to removal of each node of the tree structure association. For example, the commercial solver(s) 308 use(s) the panelist data where the first subset Lagrange multipliers are used for the equations corresponding to union removals. For example, the commercial solver(s) 308 sets of a system of equations including AUD(T) – AUD(T\1) = 0.2 for node 1 (e.g., margin A), AUD(T) – AUD(T\2) =0.4 for node 2 (e.g., margin B), . . . , and AUD(T) – AUD(T\6) = ? for node 6 (e.g., union CD).

[0076] At block 604, the example commercial solver(s) 308 select(s) value(s) for the variable Lagrange multiplier(s) (e.g., λ_0, λ_1, λ_2, λ_3, and λ_4) that satisfies the equations while using the first subset of the Lagrange multiplier(s) (e.g., λ_5 = -2.0149, λ_6 = -1.19392, λ_7 = 0.81093). As described above, each of the selected equations corresponds to Equations 2 and 4, which includes a variable s and a variable $s^{(new)}$ that can be identified using equations 3 and 5, respectively. Because Equations 2-5 include z values and $z_i = e^{\lambda i}$, the commercial solver(s) 308 can select a value for the variable Lagrange multipliers (λi)

that satisfies the select equation(s). Once the commercial solver(s) 308 select(s) the value(s) of the variable Lagrange multipliers, the example commercial solver(s) 308 determine(s) if the selected value(s) satisfy(ies) the constraint(s) of the selected equation(s) (e.g., within a threshold amount of error) (block 606).

[0077] If the example commercial solver(s) 308 determine(s) that the selected value(s) do(es) not satisfy the constraint(s) of the selected equation(s) (block 606: NO), the commercial solver(s) 308 adjust(s) the selected value(s) of the variable Lagrange multipliers (block 608) and the control returns to block 512 until the selected value(s) do(es) satisfy the constraints (e.g., within the threshold amount of error). In this manner, the example commercial solver(s) 308 performs parallel computations with a processor to determine the variable Lagrange multipliers in an iterative fashion in parallel. If the example commercial solver(s) 308 determine(s) that the selected value(s) do(es) satisfy the constraint(s) of the selected equation(s) (block 606: YES), the example process of FIG. 6 ends and control returns to a calling function or process such as a process implemented by the flowchart of FIG. 6.

[0078] FIG. 7 is an example flowchart 414 representative of example machine readable instructions that may be executed by the example union population reach determiner 120 of FIGS. 1 and/or 3 to estimate population reach(es) for union(s) based on the determined Lagrange multipliers (e.g., $\hat{\lambda}_0 = -3.17489$, $\hat{\lambda}_1 = -0.498689$, $\hat{\lambda}_2 = 1.12924$, $\hat{\lambda}_3 = -0.574063$, $\hat{\lambda}_4 = 1.7578$, $\hat{\lambda}_5 = -2.0149$, $\hat{\lambda}_6 = -1.19392$, and $\hat{\lambda}_7 = 0.81093$). The example instructions of FIG. 7 may be used to implement block 414 of FIG. 4. The example flowchart 414 is described in conjunction with the example marginal and union data 200 of FIG. 2A. However, the example flowchart 414 may be described in conjunction with any panelist data, census data, margins, and/or unions.

[0079] At block 702, the example commercial solver(s) 308 determine(s) the total population audience (e.g., AUD(T)). As described above in conjunction with Equation 2, $AUD(T) = z_0 z_N s_N$, where $z_0 z_N$ correspond to the Lagrange multipliers determined in block 412 and s_N can be solved using Equation 3. Accordingly, the commercial solver(s) 308, using Equation 2, determine(s) that since N=7, $s_1=s_2=s_3=s_4=1$, $s_5 =$

$(1+s_1\hat{z}_1)(1+s_2\hat{z}_2)-1 = 5.57925$, and $s_6 = (1+s_3\hat{z}_3)(1+s_3\hat{z}_3)-1 = 17.87222$. Accordingly, the example commercial solver(s) 308 determine(s) that $s_7 = (1+s_5\hat{z}_5)(1+s_6\hat{z}_6)-1 = 10.1885552$. Thus, the example commercial solver(s) 308 determine(s) that AUG(T) = (0.418)(2.25)(10.188552) = 0.9582.

[0080] At block 704, the example commercial solver(s) 308 select(s) an available unknown population union audience. For example, in the example of FIG. 2A, percentages corresponding to the AB, CD, and ABCD unions are unknown for the census data. Accordingly, the example commercial solver(s) 308 may select(s) the AB (e.g., corresponding to node 5 of the example tree structure 202 of FIG. 2A). At block 706, the example commercial solver(s) 308 determine(s) the total population audience percentage if the selected population union were removed from the tree structure association (e.g., AUD(T\5)). For example, using the above Equation 4, the example commercial solver(s) 308 determine(s) that AUD(T\5) = $z_0z_7s_7^{(new)}$. Because the z values are known (e.g., based on the determined Lagrange multipliers from blocks 406 and 414), to solve for AUD(T/5), the commercial solver(s) 308 utilize(s) Equation 5 to solve for $s_7^{(new)}$. For example, based on the tree structure association and Equation 5, $s_7^{(new)} = (\frac{s_7+1}{1+s_5z_5}) - 1$. Accordingly, the example commercial solver(s) 308 determine(s) that the $s_7^{(new)} = \left(\frac{(10.188552)+1}{1+(5.57925)(0.1333)}\right) - 1 = 5.4158$. Thus, the commercial solver(s) 308 determine(s) that AUD(T\5) = (0.0417986)(2.25)(5.4158) =0.5093.

[0081] At block 708, the example commercial solver(s) 308 subtract(s) the determined total population audience with the removed population union from the total population audience to determine the population union audience percentage (e.g., corresponding to Equation 1). For example, the commercial solver(s) 308 determines the total deduplicated population audience percentage of union AB to be 0.4486 (e.g., 0.4486 = 0.9582-0.5093). At block 710, the example commercial solver(s) 308 determine(s) the population reach across the selected union by multiplying the population union audience percentage by the universe estimate. For example, if the

universe estimate is 100,000 people, then the example commercial solver(s) 308 determine(s) the population reach for union AB to be 44,860 people. At block 712, the example commercial solver(s) 308 determine if there are additional unknown population union audiences to solve for. If the example commercial solvers) 308 determine that there are additional population reach(es) for additional union(s) to solve for (block 712: YES), the control returns to block 704 to determine an additional population reach for an additional union. If the example commercial solvers) 308 determine that there are not additional population reach(es) for additional union(s) to solve for (block 712: NO), the example process of FIG. 7 ends and control returns to a calling function or process such as the process implemented by FIG. 4.

[0082] FIG. 8 is a block diagram of an example processor platform 800 structured to execute the instructions of FIGS 4-7 to implement the population reach determiner 120 of FIG. 3. The processor platform 800 can be, for example, a server, a personal computer, a workstation, a self-learning machine (e.g., a neural network), a mobile device (e.g., a cell phone, a smart phone, a tablet such as an iPad™), a personal digital assistant (PDA), an Internet appliance, or any other type of computing device.

[0083] The processor platform 800 of the illustrated example includes a processor 812. The processor 812 of the illustrated example is hardware. For example, the processor 812 can be implemented by one or more integrated circuits, logic circuits, microprocessors, GPUs, DSPs, or controllers from any desired family or manufacturer. The hardware processor may be a semiconductor based (e.g., silicon based) device. In this example, the processor 812 implements the example data converter 302, the example association controller 304, and the example commercial solver(s) 308.

[0084] The processor 812 of the illustrated example includes a local memory 813 (e.g., a cache). In FIG. 8, the example local memory 813 implements the example local memory 306. The processor 812 of the illustrated example is in communication with a main memory including a volatile memory 814 and a non-volatile memory 816 via a bus 818. The volatile memory 814 may be implemented by Synchronous Dynamic Random Access Memory (SDRAM), Dynamic Random Access Memory (DRAM), RAMBUS®

Dynamic Random Access Memory (RDRAM®) and/or any other type of random access memory device. The non-volatile memory 816 may be implemented by flash memory and/or any other desired type of memory device. Access to the main memory 814, 816 is controlled by a memory controller.

[0085] The processor platform 800 of the illustrated example also includes an interface circuit 820. The interface circuit 820 may be implemented by any type of interface standard, such as an Ethernet interface, a universal serial bus (USB), a Bluetooth® interface, a near field communication (NFC) interface, and/or a PCI express interface. In the example of FIG. 8, the interface 820 implements the example interface(s) 300.

[0086] In the illustrated example, one or more input devices 822 are connected to the interface circuit 820. The input device(s) 822 permit(s) a user to enter data and/or commands into the processor 812. The input device(s) can be implemented by, for example, an audio sensor, a microphone, a camera (still or video), a keyboard, a button, a mouse, a touchscreen, a track-pad, a trackball, isopoint and/or a voice recognition system.

[0087] One or more output devices 824 are also connected to the interface circuit 820 of the illustrated example. The output devices 824 can be implemented, for example, by display devices (e.g., a light emitting diode (LED), an organic light emitting diode (OLED), a liquid crystal display (LCD), a cathode ray tube display (CRT), an in-place switching (IPS) display, a touchscreen, etc.), a tactile output device, a printer and/or speaker. The interface circuit 820 of the illustrated example, thus, typically includes a graphics driver card, a graphics driver chip and/or a graphics driver processor.

[0088] The interface circuit 820 of the illustrated example also includes a communication device such as a transmitter, a receiver, a transceiver, a modem, a residential gateway, a wireless access point, and/or a network interface to facilitate exchange of data with external machines (e.g., computing devices of any kind) via a network 826. The communication can be via, for example, an Ethernet connection, a

digital subscriber line (DSL) connection, a telephone line connection, a coaxial cable system, a satellite system, a line-of-site wireless system, a cellular telephone system, etc.

[0089] The processor platform 800 of the illustrated example also includes one or more mass storage devices 828 for storing software and/or data. Examples of such mass storage devices 828 include floppy disk drives, hard drive disks, compact disk drives, Blu-ray disk drives, redundant array of independent disks (RAID) systems, and digital versatile disk (DVD) drives.

[0090] Machine executable instructions 832 represented by FIGS. 4-7 may be stored in the mass storage device 828, in the volatile memory 814, in the non-volatile memory 816, and/or on a removable non-transitory computer readable storage medium such as a CD or DVD.

[0091] From the foregoing, it should be appreciated that the above disclosed methods, apparatus, and articles of manufacture estimate population reach from marginal ratings. Example disclosed herein determine the reach analytically using the above Equations 1-8. Traditional techniques for determining reach across various unions include determining the reach numerically. However, such traditional techniques are unsolvable for a large number of marginals dues to memory and/or processing constraints. Examples disclosed herein alleviate the problems associated with such traditional techniques by demining the reach analytically (e.g., via solving the disclosed Equations 2-8 using a population reach union estimates) in a manner that facilitates parallel processing. The parallel processing results in a faster more efficient calculation. In this manner, the estimations of reach can be determined in a faster, more efficient manner that requires less memory than traditional numerical techniques. Using examples disclosed herein reach can be determined from a nearly infinite number of instances and/or unions of media exposure based on the marginal media exposure data.

[0092] Although certain example methods, apparatus and articles of manufacture have been described herein, the scope of coverage of this patent is not limited thereto. On the contrary, this patent covers all methods, apparatus and articles of manufacture

fairly falling within the scope of the appended claims either literally or under the doctrine of equivalents.

What Is Claimed Is:

1. An apparatus comprising:

an association controller to generate a tree association corresponding to a union of a first and second margin of time; and

one or more commercial solvers to:

perform parallel computations on a processor to determine first multipliers corresponding to a total number of panelists exposed to media at the first margin, the second margin, and the union by solving first equations corresponding to the tree association;

discard a first multiplier and a second multiplier of the first multipliers corresponding to the first margin and the second margin;

perform parallel computation on the processor to determine second multipliers corresponding to a total audience exposed to the media at the first margin and the second margin by solving second equations corresponding to the tree association, the second equations including a subset of the first multipliers; and

determine an estimate for a population reach of the union based on the second multipliers.

2. The apparatus of claim 1, wherein the association controller is to generate the tree association by:

tagging the first margin with a first identifier, the second margin with a second identifier, and a third margin with a third identifier; and

tagging the first margin, the second margin, and the union with at least one of corresponding descendants or corresponding ancestors.

3. The apparatus of claim 1, wherein the first equations include a first equation corresponding to the first margin, a second equation corresponding to a second margin, and a third equation corresponding to the union.

4. The apparatus of claim 3, wherein the one or more commercial solvers are to solve for the first multiplier of the first equation and the second multiplier of the second equation in parallel.

5. The apparatus of claim 1, wherein the one or more commercial solvers are to solve the first equations iteratively.

6. The apparatus of claim 1, wherein the one or more commercial solvers are to:

keep a third multiplier of the first multipliers, the third multiplier corresponding to the union; and

include the kept third multiplier in the second equations.

7. The apparatus of claim 1, wherein the union is a first union, the tree association corresponds to a second union of a third and fourth margin of time, a total audience of the media corresponds to a third union of the first and second unions.

8. The apparatus of claim 7, wherein the one or more commercial solvers are to determine the estimate of the population reach of the first union based on a difference between a first percentage of the total audience and a second percentage of the total audience if the first union is removed from the tree association.

9. A non-transitory computer readable storage medium comprising instructions which, when executed, cause a processor to at least:

generate a tree association corresponding to a union of a first and second margin of time; and

perform parallel computations on the processor to determine first multipliers corresponding to a total number of panelists exposed to media at the first margin, the second margin, and the union by solving first equations corresponding to the tree association;

discard a first multiplier and a second multiplier of the first multipliers corresponding to the first margin and the second margin;

perform parallel computation on the processor to determine second multipliers corresponding to a total audience exposed to the media at the first margin and the second

margin by solving second equations corresponding to the tree association, the second equations including a subset of the first multipliers; and

determine an estimate for a population reach of the union based on the second multipliers.

10. The computer readable storage medium of claim 9, wherein the instructions cause the processor to generate the tree association by:

tagging the first margin with a first identifier, the second margin with a second identifier, and a third margin with a third identifier; and

tagging the first margin, the second margin, and the union with at least one of corresponding descendants or corresponding ancestors.

11. The computer readable storage medium of claim 9, wherein the first equations include a first equation corresponding to the first margin, a second equation corresponding to a second margin, and a third equation corresponding to the union.

12. The computer readable storage medium of claim 11, wherein the instructions cause the processor to solve for the first multiplier of the first equation and the second multiplier of the second equation in parallel.

13. The computer readable storage medium of claim 9, wherein the instructions cause the processor to solve the first equations iteratively.

14. The computer readable storage medium of claim 9, wherein the instructions cause the processor to:

keep a third multiplier of the first multipliers, the third multiplier corresponding to the union; and

include the kept third multiplier in the second equations.

15. The computer readable storage medium of claim 9, wherein the union is a first union, the tree association corresponds to a second union of a third and fourth margin of time, a total audience of the media corresponds to a third union of the first and second unions.

16. The computer readable storage medium of claim 15, wherein the instructions cause the processor to determine the estimate of the population reach of the

first union based on a difference between a first percentage of the total audience and a second percentage of the total audience if the first union is removed from the tree association.

17. A method comprising:

generating a tree association corresponding to a union of a first and second margin of time; and

performing parallel computations on a processor to determine first multipliers corresponding to a total number of panelists exposed to media at the first margin, the second margin, and the union by solving first equations corresponding to the tree association;

discarding a first multiplier and a second multiplier of the first multipliers corresponding to the first margin and the second margin;

performing parallel computation on the processor to determine second multipliers corresponding to a total audience exposed to the media at the first margin and the second margin by solving second equations corresponding to the tree association, the second equations including a subset of the first multipliers; and

determining an estimate for a population reach of the union based on the second multipliers.

18. The method of claim 17, wherein the generating of the tree association includes:

tagging the first margin with a first identifier, the second margin with a second identifier, and a third margin with a third identifier; and

tagging the first margin, the second margin, and the union with at least one of corresponding descendants or corresponding ancestors.

19. The method of claim 17, wherein the first equations include a first equation corresponding to the first margin, a second equation corresponding to a second margin, and a third equation corresponding to the union.

20. The method of claim 19, further including solving for the first multiplier of the first equation and the second multiplier of the second equation in parallel.

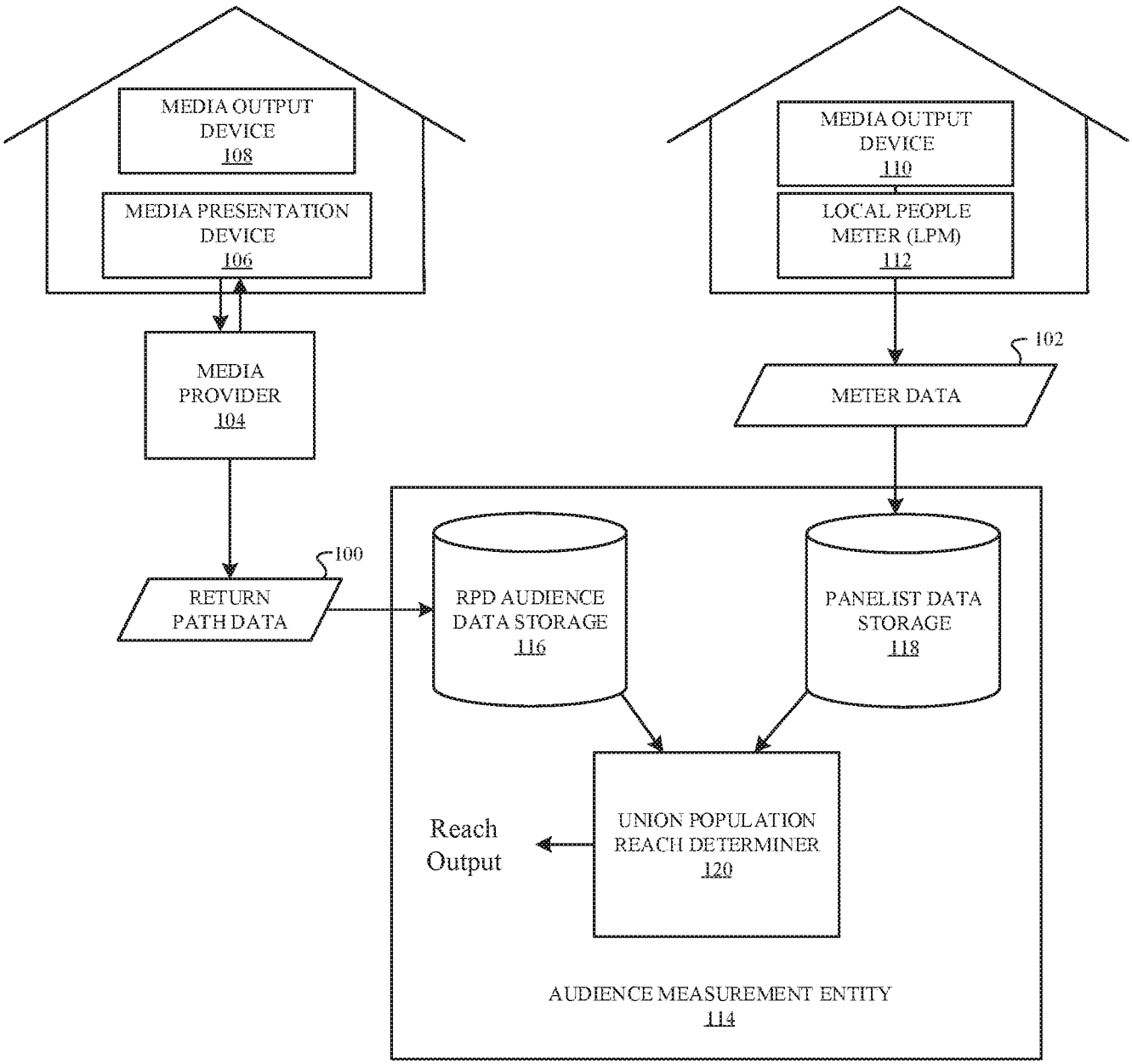

MEDIA OUTPUT DEVICE 108
MEDIA PRESENTATION DEVICE 106
MEDIA PROVIDER 104
MEDIA OUTPUT DEVICE 110
LOCAL PEOPLE METER (LPM) 112
102
METER DATA
100
RETURN PATH DATA
RPD AUDIENCE DATA STORAGE 116
PANELIST DATA STORAGE 118
Reach Output
UNION POPULATION REACH DETERMINER 120
AUDIENCE MEASUREMENT ENTITY 114

FIG. 1

200

Union/Margin	Panel	Census
A	0.1	0.2
B	0.2	0.4
C	0.3	0.6
D	0.4	0.8
(AB)	0.25	?
(CD)	0.55	?
(ABCD)	0.7	?
UE=1.0		

FIG. 2A

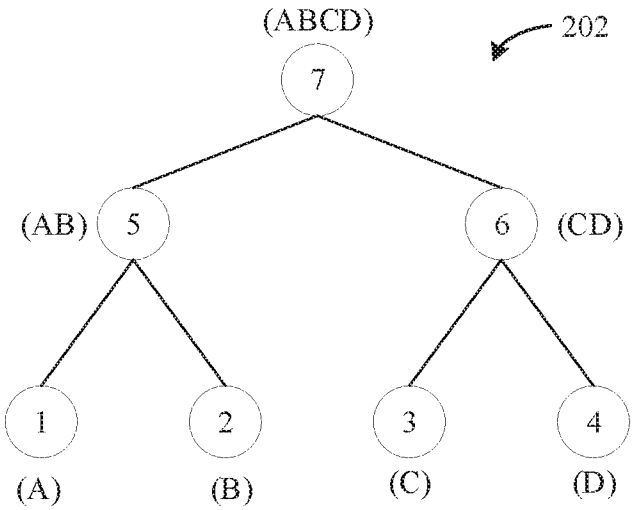


FIG. 2B

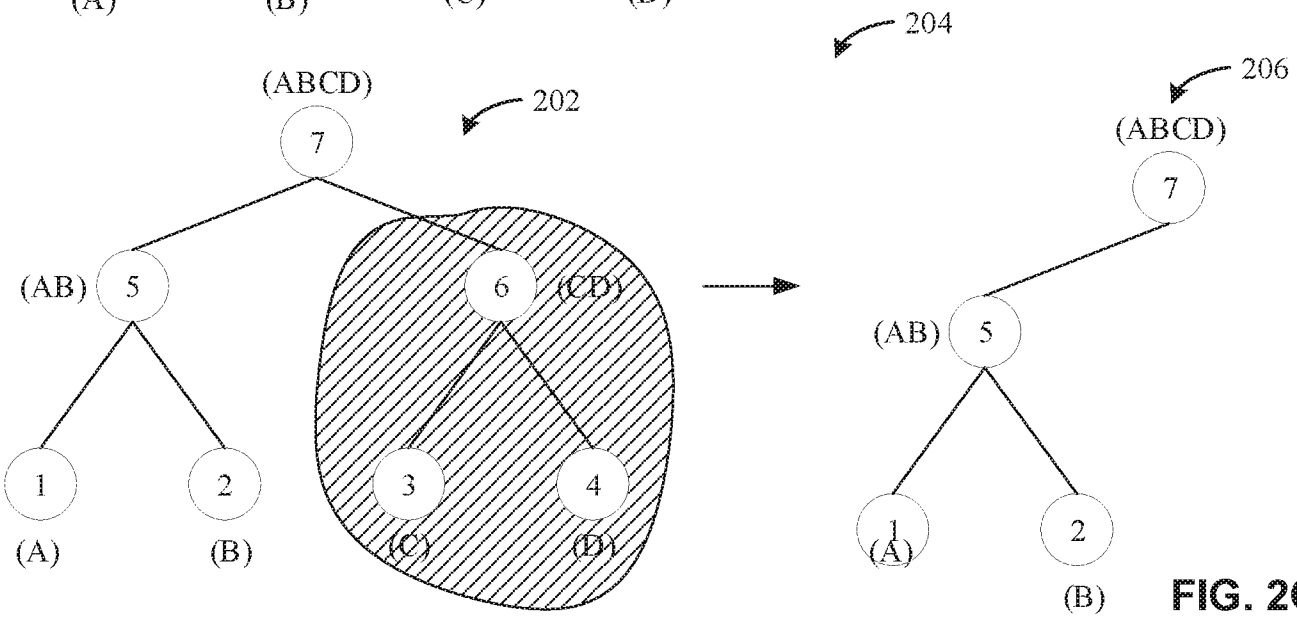


FIG. 2C

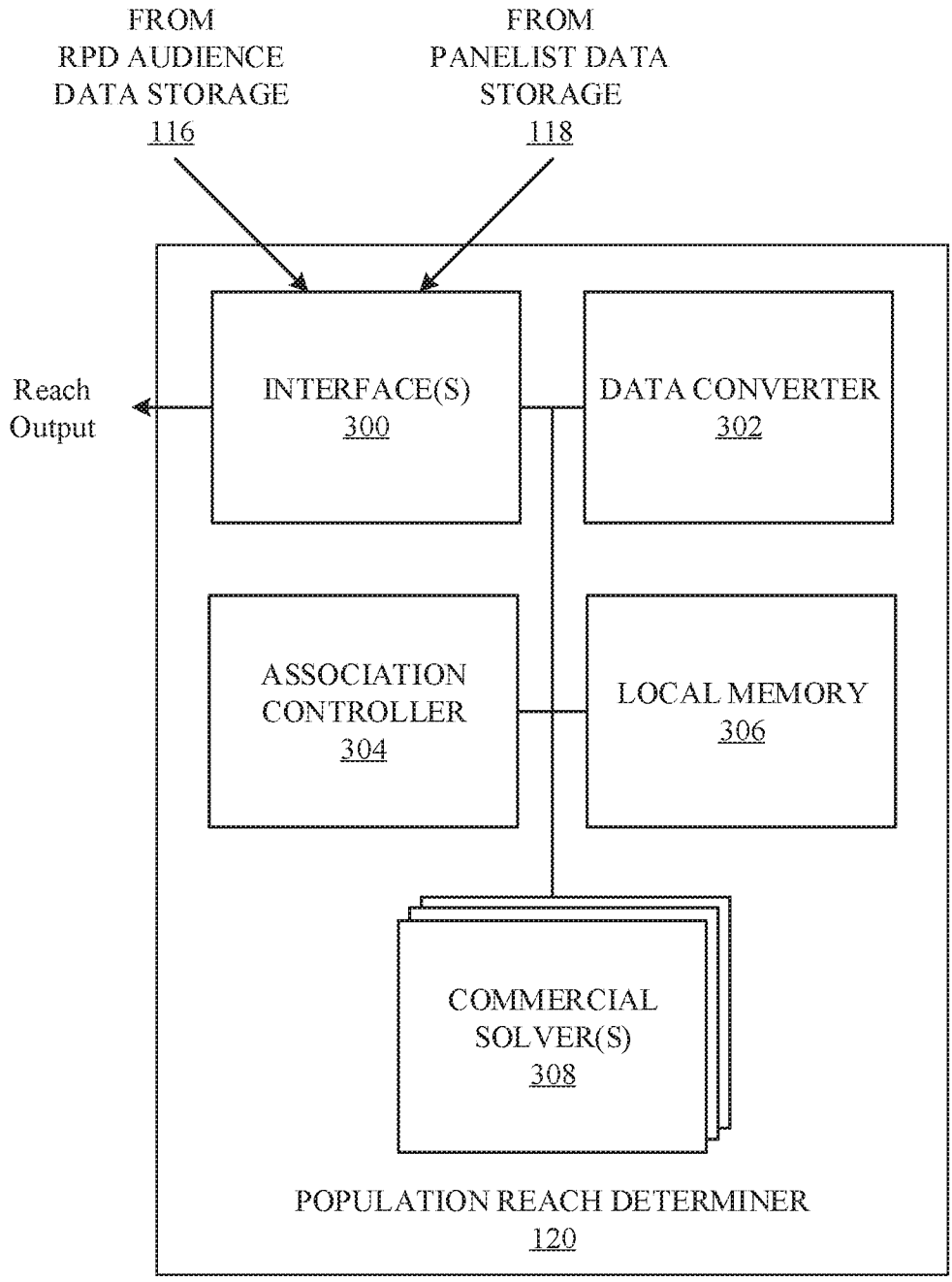

FROM
RPD AUDIENCE
DATA STORAGE
116
FROM
PANELIST DATA
STORAGE
118
Reach
Output
INTERFACE(S)
300
DATA CONVERTER
302
ASSOCIATION
CONTROLLER
304
LOCAL MEMORY
306
COMMERCIAL
SOLVER(S)
308
POPULATION REACH DETERMINER
120

FIG. 3

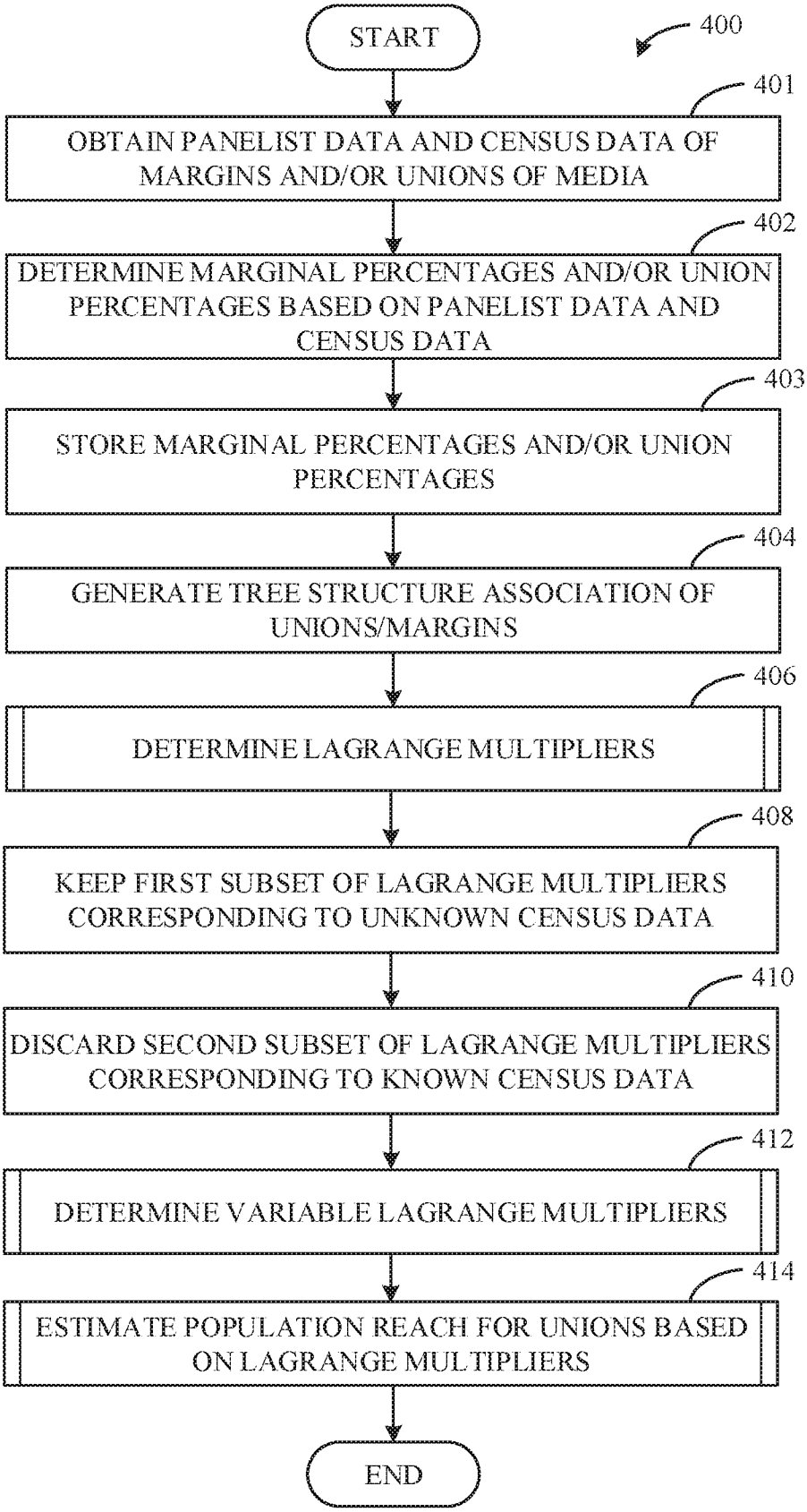

START
400
401
OBTAIN PANELIST DATA AND CENSUS DATA OF MARGINS AND/OR UNIONS OF MEDIA
402
DETERMINE MARGINAL PERCENTAGES AND/OR UNION PERCENTAGES BASED ON PANELIST DATA AND CENSUS DATA
403
STORE MARGINAL PERCENTAGES AND/OR UNION PERCENTAGES
404
GENERATE TREE STRUCTURE ASSOCIATION OF UNIONS/MARGINS
406
DETERMINE LAGRANGE MULTIPLIERS
408
KEEP FIRST SUBSET OF LAGRANGE MULTIPLIERS CORRESPONDING TO UNKNOWN CENSUS DATA
410
DISCARD SECOND SUBSET OF LAGRANGE MULTIPLIERS CORRESPONDING TO KNOWN CENSUS DATA
412
DETERMINE VARIABLE LAGRANGE MULTIPLIERS
414
ESTIMATE POPULATION REACH FOR UNIONS BASED ON LAGRANGE MULTIPLIERS
END

FIG. 4

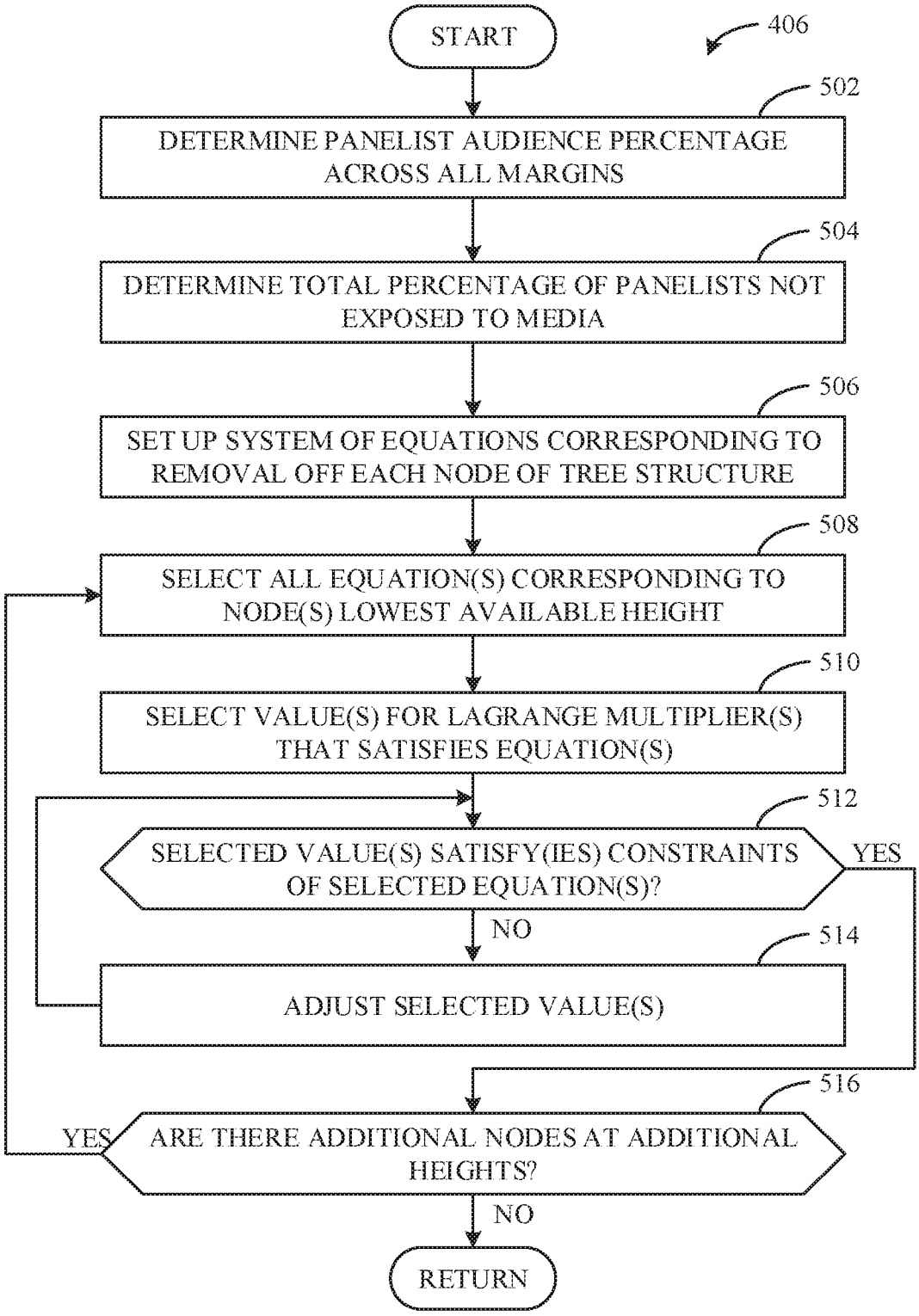

START
406
502
DETERMINE PANELIST AUDIENCE PERCENTAGE ACROSS ALL MARGINS
504
DETERMINE TOTAL PERCENTAGE OF PANELISTS NOT EXPOSED TO MEDIA
506
SET UP SYSTEM OF EQUATIONS CORRESPONDING TO REMOVAL OFF EACH NODE OF TREE STRUCTURE
508
SELECT ALL EQUATION(S) CORRESPONDING TO NODE(S) LOWEST AVAILABLE HEIGHT
510
SELECT VALUE(S) FOR LAGRANGE MULTIPLIER(S) THAT SATISFIES EQUATION(S)
512
SELECTED VALUE(S) SATISFY(IES) CONSTRAINTS OF SELECTED EQUATION(S)?
YES
NO
514
ADJUST SELECTED VALUE(S)
516
YES
ARE THERE ADDITIONAL NODES AT ADDITIONAL HEIGHTS?
NO
RETURN

FIG. 5

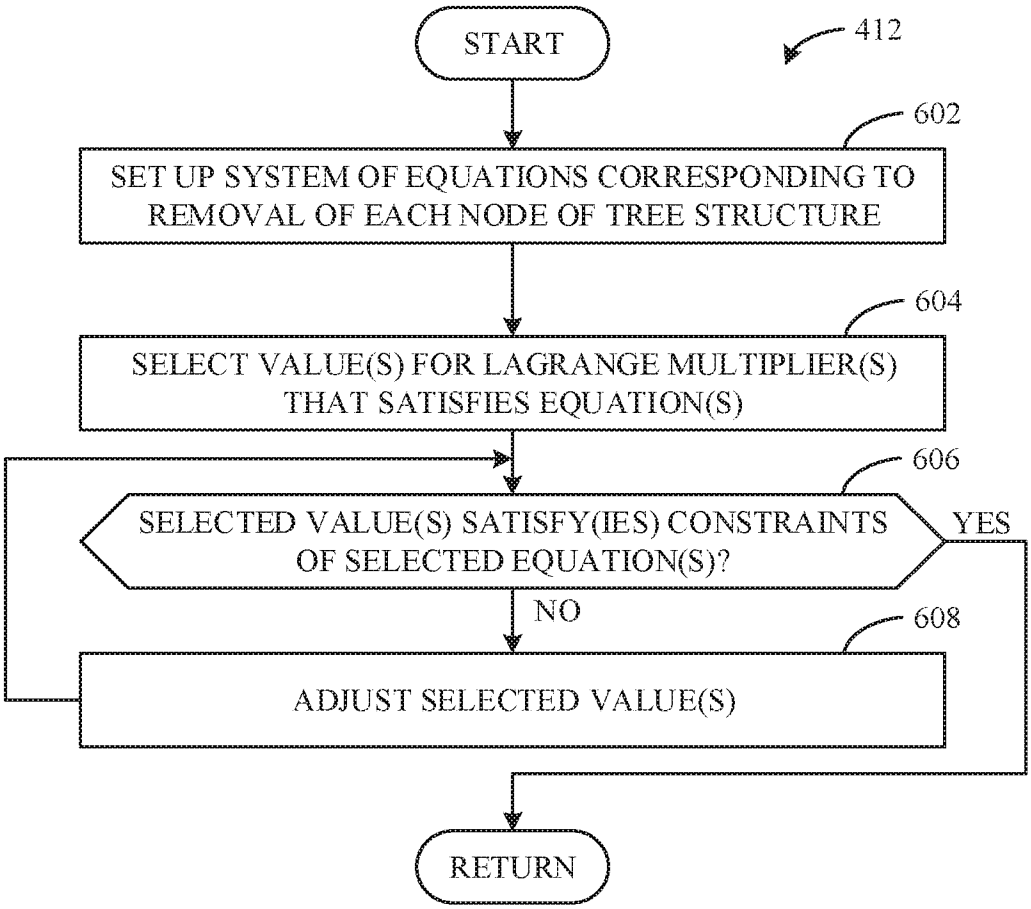

START
412
602
SET UP SYSTEM OF EQUATIONS CORRESPONDING TO REMOVAL OF EACH NODE OF TREE STRUCTURE
604
SELECT VALUE(S) FOR LAGRANGE MULTIPLIER(S) THAT SATISFIES EQUATION(S)
606
SELECTED VALUE(S) SATISFY(IES) CONSTRAINTS OF SELECTED EQUATION(S)?
YES
NO
608
ADJUST SELECTED VALUE(S)
RETURN

FIG. 6

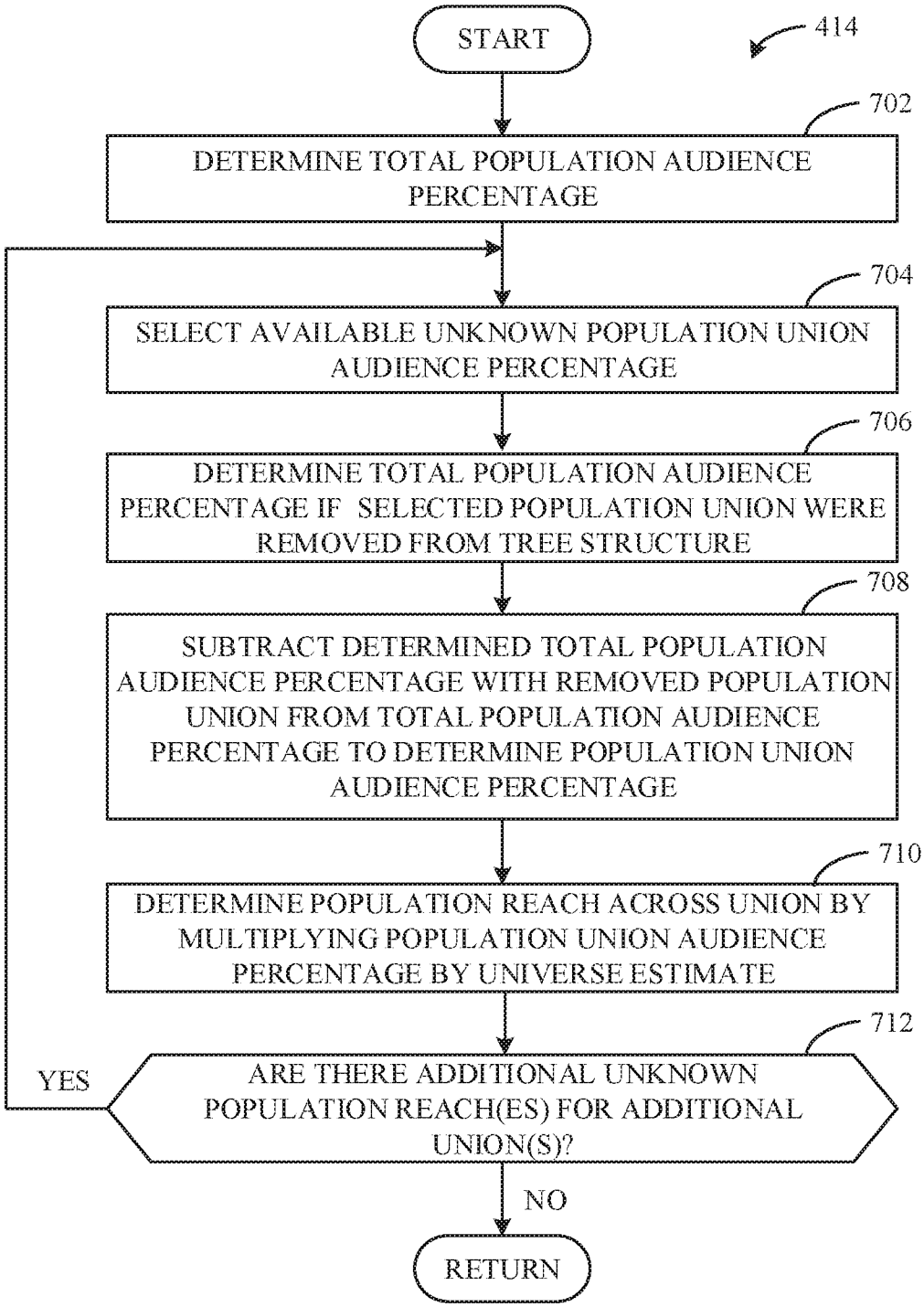

START
414
702
DETERMINE TOTAL POPULATION AUDIENCE PERCENTAGE
704
SELECT AVAILABLE UNKNOWN POPULATION UNION AUDIENCE PERCENTAGE
706
DETERMINE TOTAL POPULATION AUDIENCE PERCENTAGE IF SELECTED POPULATION UNION WERE REMOVED FROM TREE STRUCTURE
708
SUBTRACT DETERMINED TOTAL POPULATION AUDIENCE PERCENTAGE WITH REMOVED POPULATION UNION FROM TOTAL POPULATION AUDIENCE PERCENTAGE TO DETERMINE POPULATION UNION AUDIENCE PERCENTAGE
710
DETERMINE POPULATION REACH ACROSS UNION BY MULTIPLYING POPULATION UNION AUDIENCE PERCENTAGE BY UNIVERSE ESTIMATE
712
YES
ARE THERE ADDITIONAL UNKNOWN POPULATION REACH(ES) FOR ADDITIONAL UNION(S)?
NO
RETURN

FIG. 7

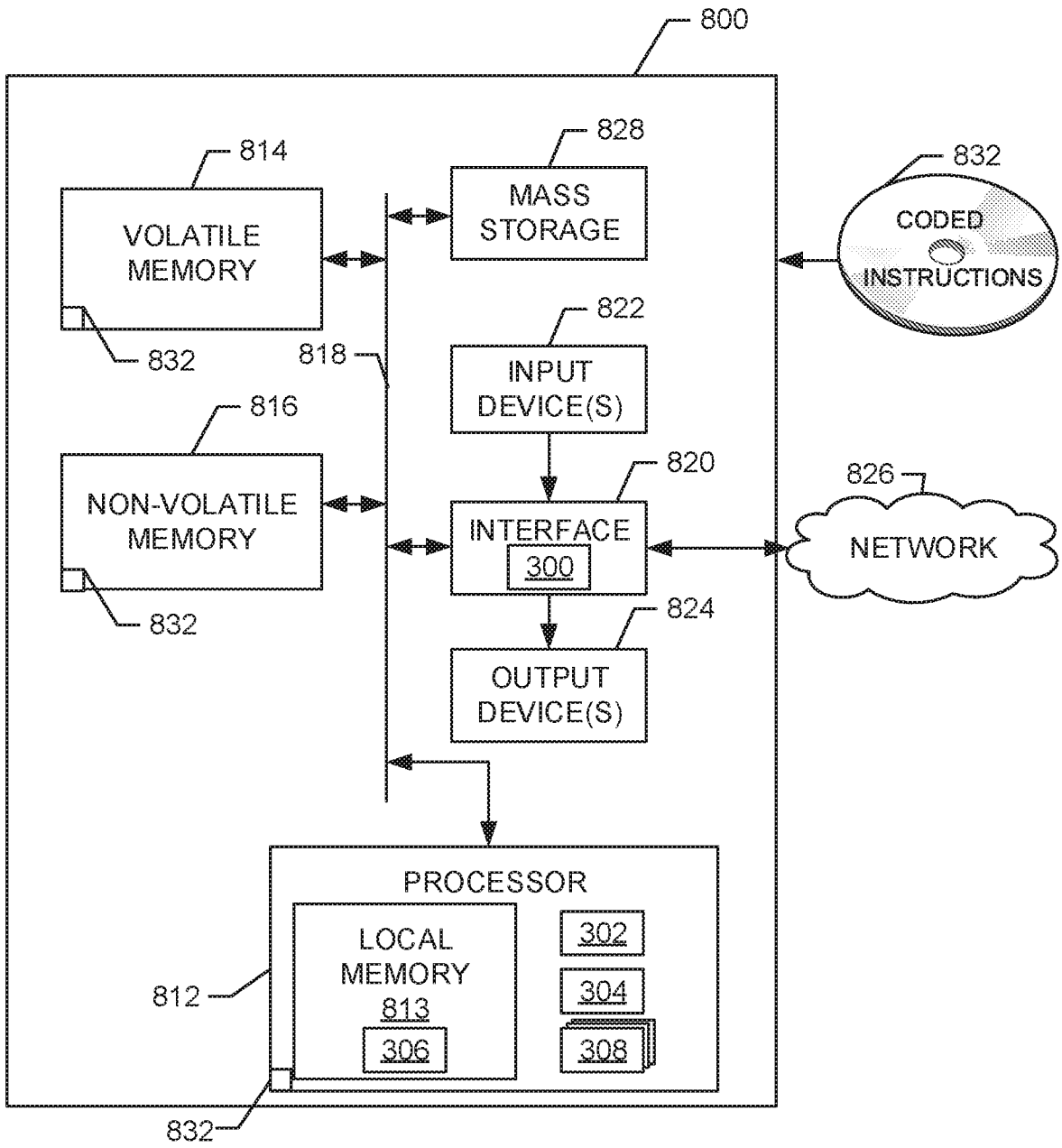

800
814
VOLATILE MEMORY
832
816
NON-VOLATILE MEMORY
832
818
828
MASS STORAGE
822
INPUT DEVICE(S)
820
INTERFACE
300
824
OUTPUT DEVICE(S)
832
CODED INSTRUCTIONS
826
NETWORK
PROCESSOR
812
LOCAL MEMORY
813
306
302
304
308
832

FIG. 8

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US2020/022436

A. CLASSIFICATION OF SUBJECT MATTER

H04H 60/33(2008.01)i, H04H 60/63(2008.01)i, G06F 17/11(2006.01)i

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H04H 60/33; G06F 17/10; G06Q 30/02; G06Q 50/00; H04H 60/32; H04N 21/24; H04N 21/262; H04H 60/63; G06F 17/11

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
Korean utility models and applications for utility models
Japanese utility models and applications for utility models

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
eKOMPASS(KIPO internal) & Keywords: tree, population, reach, margin

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 2016-0165277 A1 (GOOGLE INC.) 09 June 2016 paragraphs [0069]-[0078]; and claims 1-9	1-20
A	US 8087041 B2 (QI FU et al.) 27 December 2011 column 11, line 1 - column 12, line 33; and claims 1-20	1-20
A	US 2013-0346033 A1 (JIANQIANG WANG et al.) 26 December 2013 paragraphs [0019]-[0022]; and claims 1-8	1-20
A	US 9070139 B2 (HULU, LLC) 30 June 2015 column 1, line 51 - column 2, line 50; and claims 1-7	1-20
A	US 2015-0332310 A1 (FACEBOOK, INC.) 19 November 2015 paragraphs [0004]-[0010]; and claims 1-17	1-20

☐ Further documents are listed in the continuation of Box C.

☒ See patent family annex.

* Special categories of cited documents:
"A" document defining the general state of the art which is not considered to be of particular relevance
"D" document cited by the applicant in the international application
"E" earlier application or patent but published on or after the international filing date
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed
"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents,such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
03 July 2020 (03.07.2020)	**06 July 2020 (06.07.2020)**

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INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No.

PCT/US2020/022436

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US 2016-0165277 A1	09/06/2016	None	
US 8087041 B2	27/12/2011	EP 2366227 A2	21/09/2011
		US 2010-0146531 A1	10/06/2010
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		WO 2010-068658 A2	17/06/2010
		WO 2010-068658 A3	05/05/2011
US 2013-0346033 A1	26/12/2013	None	
US 9070139 B2	30/06/2015	US 2014-0081767 A1	20/03/2014
US 2015-0332310 A1	19/11/2015	None	